Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Mathematical Psychology 49 (2005) 383–402
Journal of Mathematical Psychology
www.elsevier.com/locate/jmp

Associative isolation: Unifying associative and list memory

Jeremy B. Caplan*

The Rotman Research Institute, Baycrest Centre for Geriatric Care, Toronto, Ont., Canada M6A 2E1

Received 21 November 2002; received in revised form 3 May 2005
Available online 25 July 2005

Abstract

Rather than treating paired associate and serial learning as involving the acquisition of distinct types of information [e.g. Murdock (1974). *Human memory*: *Theory and data*. New York: Wiley] I propose an Isolation Principle which treats the two as ends of a continuum. According to this principle, consecutive pairs of items are relatively isolated from other studied items in paired associates learning, but not isolated in serial list learning. The consequence is that variability that dominates forward and backward probed recall is highly correlated in pairs but less so, due to differential interference, in lists. This can explain an important dissociation: whereas forward and backward probes of pairs are nearly perfectly correlated, the correlation is only moderate for serial lists. I demonstrate this in a chaining model by varying item-to-item associative strengths and in a positional coding model by varying the similarity structure of item positions. This enables a range of models to account for data on pairs and lists, as well as potential intermediate or hybrid paradigms, within a single theoretical framework.

Keywords: Associative symmetry; Paired associates learning; Serial order memory; Associative chaining; Positional coding

1. Introduction

1.1. Associative and order information

Memory researchers have looked to paired associates learning (PAL) paradigms to understand associative memory, and to serial learning (SL) paradigms to understand memory for order. However, Ebbinghaus (1885/1913) proposed that serial order memory has an associative basis. Formalized by Lewandowsky and Murdock (1989), this class of *associative chaining* models build ordered lists by linking together nearest-neighbor items (and sometimes non-adjacent items) into pairwise associative units. In contrast, the other class of models of serial order memory, which we term *positional coding*[1] models (e.g., Brown et al., 2000; Burgess and Hitch, 1999; Henson, 1998; Lee and Estes, 1977),treat only serial list learning, thus tacitly implying that associative and list memory need to be treated separately.

First I discuss two apparent dissociations between PAL and SL paradigms. Then I introduce the Isolation Principle as a way of treating PAL and SL as ends of a continuum, enabling a broad class of models to account for data on PAL, SL and potentially, a range of intermediate paradigms.

*Fax: +416 785 2862.
E-mail address: jcaplan@rotman-baycrest.on.ca.

[1]Several modellers have distinguished between positional and order coding models, position being absolute while order being relative. While this is a valid distinction, it is not critical to the theoretical points being made here. Ultimately, one must address order-coding models separately from positional-coding models. However, a more basic distinction is whether items are associated directly to one another, as in the chaining class of models, or interact indirectly via a separate, ordered structure, be it positional, order or something more abstract like temporal context. Thus, I refer to such models as "positional" coding models as a short-hand to emphasize this fundamental distinction between model classes.

0022-2496/$ - see front matter
doi:10.1016/j.jmp.2005.06.004

1.2. PAL: associative symmetry

Early research on PAL stemmed from stimulus–response theory, so the prevailing assumption was that paired associates learning possessed intrinsic directionality. Backward associations were rarely discussed, and some (e.g., Ebbinghaus, 1885/1913; Wolford, 1971) even proposed that forward and backward associations are independent. Opposed to this notion, Köhler (1947), and later, Asch and Ebenholtz (1962), proposed the Associative Symmetry Hypothesis, stating that, instead of participants learning two separate directional associations, paired items are encoded into a single, holistic representation, with no meaningful distinction between the forward and backward associations. They proposed that asymmetries in measured behavior were the result of asymmetries the experimental paradigms. Certain experimental variables must be controlled for to test this notion, including differences in type of material (e.g., adjectives versus nouns) between "A" and "B" items from the "A–B" pair and "response availability," referring to participants begin more practised at producing the "B" than the "A" items. If one equates these conditions, mean performance on forward and backward probes is symmetric, supporting Associative Symmetry (e.g., Horowitz et al., 1964; Murdock, 1962, 1965, 1966). See Kahana (2002) for a review. Taking a different approach, Horowitz et al. (1966) presented participants with double-function lists, where each item is an "A" item of one pair and a "B" item of a different pair (e.g., Primoff, 1938; Slamecka, 1976). They asked participants to give two "free associations" to probe items from the list. Participants responded with the forward item just as often, on average, as with the backward item. The exception was probing the terminal pairs (first and last pairs from the serial list from which the double-function pairs were derived), which were single-function. These findings were used as evidence that there is no intrinsic directionality in memory for pairs.

However, Kahana (2000, Chap.4) pointed out that both holistic and non-holistic models could produce symmetry or asymmetry in *mean performance*. The symmetry results could reflect that *on average*, forward and backward associations are encoded with comparable strength, but for specific pairs, one directional association could be learned without the other. Associative Symmetry specifically requires that performance on forward and backward probes of paired associates should be perfectly correlated at the level of pairs (Kahana, 2000). Using successive testing, Kahana (2002) showed empirically that performance on individual pairs is highly correlated between forward and backward probes, directly supporting Associative Symmetry. These very high (nearly perfect) correlations were obtained between successive tests that were separated from each other and from study by substantial numbers of other items.

Finally, Rizzuto and Kahana (2001) showed that a model could only account for correlated forward and backward probe performance if the encoded forward and backward associations highly correlated in strength. This held at three different levels of learning, so it could not be attributed to floor or ceiling effects in accuracy. They also compared a simulated model with and without output encoding (storing correctly recalled pairs during the first test) and found that when output encoding was included, the model still required quite highly correlated forward and backward stored associations in order to fit the data, ruling out output encoding as the primary cause of correlated probed recall.

Thus, there is strong support for associative symmetry in pairs as seen in the high forward–backward probe correlation and modelling showing that this empirical finding in behavior requires the underlying encoded associations also to be highly correlated.

1.3. SL: associative asymmetry

If serial lists are built as a chain of simple associations of the kind measured in PAL (Ebbinghaus, 1885/1913; Lewandowsky and Murdock, 1989) then one might expect to see similarly high correlations between forward and backward probes of serial lists. Conversely, the lack of such high correlations might support the view that SL requires distinct study and/or recall processes than PAL, or else that SL and PAL tap different subsets of the information that was encoded during study. Kahana and Caplan (2002) ran such a probed SL experiment. Although they observed a forward probe advantage in mean accuracy, they did not analyze their data in a way that would directly speak to the crucial question of the correlation between forward and backward probes of a given pair. I now report this re-analysis of the Kahana and Caplan (2002) data; namely, I compute the correlation between forward and backward probes of a serial list.

The experimental methods are outlined by Kahana and Caplan (2002). Here I summarize the most relevant methodological parameters and procedures. Each participant learned 20 lists of 19 words. Following serial learning of each list to a perfect recall criterion (with no overlearning trials), the participant performed a brief distractor task, followed by probed recall. The probes were sequential probes (Murdock, 1967) of pairs of items, where the pairs were originally presented in adjacent serial positions in the previously learned serial list. Participants were instructed to respond vocally both to the distractor (deciding whether an equation was true or false) and cued recall probes (with the target word) and were encouraged to respond as quickly and accurately as possible, with the knowledge that their

Table 1
Mean accuracy for forward and backward probed recall of paired associates [data from Kahana (2002)] of pairs of adjacent items in learned serial lists [data from Kahana and Caplan (2002)]

	PAL			SL
	1 pres	3 pres	5 pres	Criterion
Forward	0.329 (0.225)	0.648 (0.291)	0.732 (0.230)	0.894 (0.010)
Backward	0.336 (0.222)	0.644 (0.237)	0.718 (0.209)	0.848 (0.009)

For paired associates (PAL) data, number of presentations was manipulated within subjects; for serial learning (SL) data, lists were all first learned to a criterion of one perfect recall before probing. Standard errors (in parentheses) are corrected for between-subjects variance (Loftus and Masson, 1994).

responses and response times were being recorded. As previously reported, forward probes showed a small but significant advantage in accuracy over backward probes (Table 1, last column), in contrast to symmetric mean performance in Kahana's (2002) PAL data (Table 1, first three columns).

As is the case when mean accuracy is symmetric (i.e., in PAL), the asymmetric mean performance obtained in SL does not necessarily tell us anything about associative symmetry. Again, it is the correlation that more directly speaks to the question of associative symmetry. In a re-analysis of the probed serial list data from Kahana and Caplan (2002) I now report the correlation between forward and backward successive probes of a learned serial list. Each list was probed with seven possible cue types, where cues could involve one or two list items. Only the single-item adjacent cues (forward: **A?** and backward: **?B**) are considered here. The list was probed completely (i.e., every item was involved in exactly one probe, being either a cue item, a target item or a skipped item in the case of remote probes) and then probed completely once again, in a new random order. I label these two sets of probes Test 1 and Test 2, respectively. I define a "list transition" as two items in adjacent serial positions. In each list, two list transitions were probed with single-item adjacent cues, and if a list transition was probed on Test 1 with a single-item adjacent probe, it was probed again on Test 2 also with a single-item adjacent probe. Probe direction was chosen randomly on tests 1 and 2 (the rest of the list was probed with other, more complex types of probes, which are excluded from the present analysis). This yielded two Test 1/Test 2 data points per list, for a total of 40 data points per participant. One can measure the correlation between accuracy on forward and backward probes using Yule's $\mathcal{Q}$. Yule's $\mathcal{Q}$ is equivalent to Γ for a discrete correlation for a 2×2 contingency table (see Kahana, 2002, for a review). For each 2×2 contingency table, there are four different tallies of the possible outcomes: a: Test 1/Test 2 = Correct/Correct; b: Correct/Incorrect; c: Incorrect/Correct and d: Incorrect/Incorrect. Yule's $\mathcal{Q} = (ad - bc)/(ad + bc)$. Note that in the successive testing paradigm, Tests 1 and 2 cannot be guaranteed to be independent tests. That is, Test 1 may contaminate Test 2. For this reason it is important to compare Yule's $\mathcal{Q}$ for simple test/re-test effects, as well as obtaining some sort of lower bound to the possible obtainable value of $\mathcal{Q}$. All reported t tests are two-tailed and paired-samples except when comparing between groups of participants.[2]

I compute the average Yule's $\mathcal{Q}$ across participants for three conditions: (1) "Same Direction," condition ($\mathcal{Q}_{\mathrm{Same}}$): cases in which direction was the same on Test 1 and Test 2 (**A?/A?** or **?B/?B**), (2) "Different Direction" condition ($\mathcal{Q}_{\mathrm{FB}}$): in which probe direction was different on Test 1 and Test 2 (**A?/?B** and **?B/A?**) and (3) "Within-List Control" condition ($\mathcal{Q}_{\mathrm{Control}}$): a bootstrap resampling in which Test 1 and Test 2 performance measures were drawn from different list transitions but from the same list. This control is an estimate of Yule's $\mathcal{Q}$ produced by list-to-list variability, and was collapsed across all combinations of probe directions. Note that whereas for $\mathcal{Q}_{\mathrm{Same}}$ and $\mathcal{Q}_{\mathrm{FB}}$, the same list transition (and serial positions) are tested on Test 1 and Test 2, different list transitions are probed on Test 1 and Test 2 to compute $\mathcal{Q}_{\mathrm{Control}}$. I also recompute Yule's $\mathcal{Q}$ for each condition for Kahana's (2002) PAL data, including computing the Within-List Control. The control, $\mathcal{Q}_{\mathrm{Control}}$, was calculated by re-pairing Test 1 and Test 2 within a given list (and within a given level of *#presentations*). All such combinations were used except for combinations where Test 1 and Test 2 probe the same pair (i.e., those that were used to compute $\mathcal{Q}_{\mathrm{Same}}$ and $\mathcal{Q}_{\mathrm{FB}}$) and to avoid end-of-list artifacts, probes of positions 1, 2, 18 and 19 were excluded from all analyses. The critical measure is $\Delta\mathcal{Q} = \mathcal{Q}_{\mathrm{Same}} - \mathcal{Q}_{\mathrm{FB}}$.

Table 2 shows the mean Yule's $\mathcal{Q}$ values for each condition, for Kahana's (2002) PAL data (column 1), and for Kahana and Caplan's (2002) probed serial list data (column 2). First, for the probed SL data, $\mathcal{Q}_{\mathrm{FB}}$ is significantly lower than $\mathcal{Q}_{\mathrm{Same}}$ [$T(59) = -4.98$, $p < 0.0001$], confirming that forward and backward probed recall are not as correlated as the maximum correlation one could expect (namely, the correlation between Test 1 and Test 2 of the identical probe). $\mathcal{Q}_{\mathrm{FB}}$ is also significantly greater than $\mathcal{Q}_{\mathrm{Control}}$ [$T(59) = 4.74$, $p < 0.0001$], confirming that this correlation is not simply produced by list-to-list variability.

For the PAL data, as previously reported, both $\mathcal{Q}_{\mathrm{Same}}$ and $\mathcal{Q}_{\mathrm{FB}}$ are extremely high. These values, while close in

[2]Instead of computing a separate Yule's $\mathcal{Q}$ value for each participant one can collapse data across all subjects. When I did so, for both PAL and SL data sets the qualitative pattern of findings was preserved while the overall correlations tended to increase, as expected, due to between-subjects variability that can inflate $\mathcal{Q}$ values (Simpson's Paradox; cf. Hintzman, 1993).

Table 2
Yule's $\mathcal{Q}$ between Test 1 and Test 2 for PAL [column 1, data from Kahana (2002)] and probed SL [column 2, data from Kahana and Caplan (2002)]

	PAL	SL
Same direction	0.993 (0.002)	0.85 (0.03)
Different direction	0.928 (0.011)	0.58 (0.03)
Within-list control	0.327 (0.061)	0.35 (0.04)

Yule's $\mathcal{Q}$ for PAL are collapsed across #presentations. $\mathcal{Q}$ values are averaged over participants (PAL: $N = 15$; SL: $N = 60$). Conditions are: "Same Direction" (**A?/A?** or **?B/?B**), "Different Direction" (**A?/?B** and **?B/A?**) and "Within-List Control" (different list transitions on Test 1 and 2, regardless of direction). Standard errors (in parentheses) are corrected for between-subjects variance (Loftus and Masson, 1994).

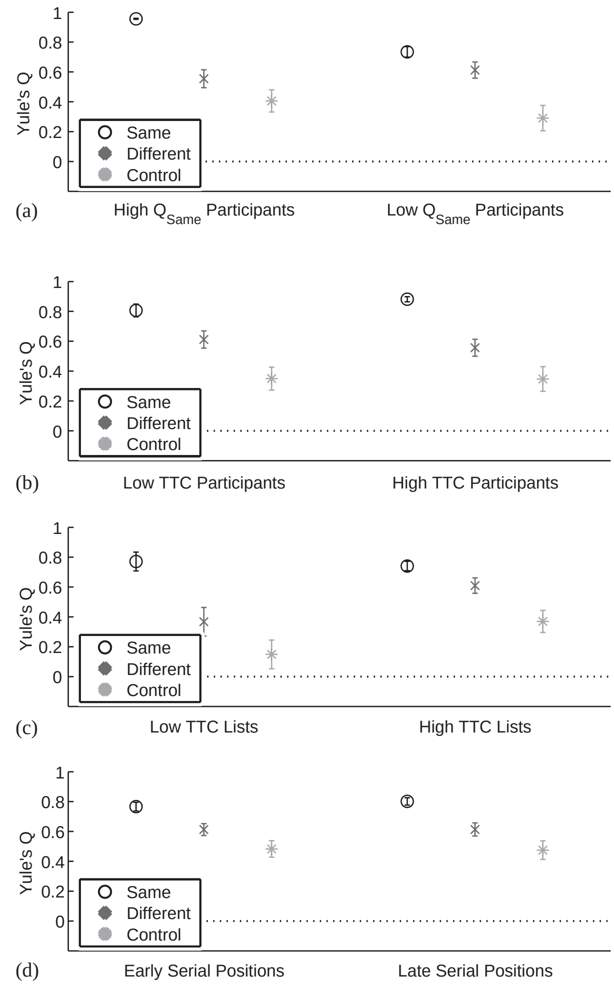

Fig. 1. Yule's $\mathcal{Q}$ between Test 1 and Test 2 for probed SL [data from Kahana and Caplan, 2002]. Conditions are: "Same" (**A?/A?** or **?B/?B**), "Different" (**A?/?B** and **?B/A?**) and "Control" (different list transitions on Test 1 and Test 2, regardless of direction). $\mathcal{Q}$ values are plotted for the following median splits: (a) participants with high versus low $\mathcal{Q}_{\text{Same}}$, (b) participants who learned lists in few versus many trials, (c) probes of lists initially learned in few versus many study–test trials and (d) probes from early versus late serial positions. Error bars are standard error of the mean across subjects and are thus conservative for the purposes of paired-samples t-tests.

magnitude, differ significantly [$T(14) = 6.2, p < 0.0001$]. Note that given that the correlations are near ceiling, their variances may be artifactually low. Rizzuto and Kahana (2001) showed that simulations of a neural network model could only fit these correlations if the correlation between the encoding strengths for forward and backward associations was nearly perfect. This held even when fitting individual participants' data, so it could not be explained by between-subjects variability. We can further compare these previously reported values to the new calculation of $\mathcal{Q}_{\text{Control}}$. Compared to this control, it is clear that both $\mathcal{Q}_{\text{Same}}$ and $\mathcal{Q}_{\text{FB}}$ values are exceptionally high [$\mathcal{Q}_{\text{FB}}$ differs highly significantly from $\mathcal{Q}_{\text{Control}}$; $T(14) = 10.6, p < 0.0001$], and that these near-perfect correlations cannot be accounted for simply by list-to-list variability, nor by variability induced by the number-of-presentations manipulation of pair strength. It should be noted that the statistical *significance* of $\mathcal{Q}_{\text{Same}}$ versus $\mathcal{Q}_{\text{FB}}$ comparison is stronger for pairs than for lists. However, this has to due with statistical power, and may be due to superficial differences between the experimental paradigms. More relevant to our discussion are the *magnitudes* of the differences between $\mathcal{Q}$ values. With respect to the mean values of $\mathcal{Q}$, the value for $\mathcal{Q}_{\text{FB}}$ in the PAL data sits near the upper end of the range set by $\mathcal{Q}_{\text{Same}}$ and $\mathcal{Q}_{\text{Control}}$. In contrast, for the SL data, $\mathcal{Q}_{\text{FB}}$ sits closer to the center of the range set by $\mathcal{Q}_{\text{Same}}$ and $\mathcal{Q}_{\text{Control}}$.

While the accuracy and correlation findings suggest a dissociation between PAL and SL, one should bear in mind not only that the comparison is across experiments, but many aspects of the experimental design differed. We can, however, test whether certain aspects of the designs that differed between experiments could have produced what appears like a dissociation between PAL and SL. I briefly address these now.

The most worrying difference between the two experiments is that the $\mathcal{Q}_{\text{Same}}$ (test/re-test correlation) is near-perfect for the PAL data set but far from perfect for SL. It is possible that the increased Test 1/Test 2 variability is somehow making it easier to observe the substantially lower $\mathcal{Q}_{\text{FB}}$ in SL, but perhaps a SL data set that had high $\mathcal{Q}_{\text{Same}}$ would show a much greater $\mathcal{Q}_{\text{FB}}$, weakening the evidence for a dissociation between PAL and SL. To test this hypothesis, I median-split the participants based on their $\mathcal{Q}_{\text{Same}}$ values. As Fig. 1a shows, the $\Delta\mathcal{Q}$ remains large for both groups of participants. This was confirmed by t-tests: $\mathcal{Q}_{\text{Same}}$ differed from $\mathcal{Q}_{\text{FB}}$ for both groups [High $\mathcal{Q}_{\text{Same}}$ Group: $T(29) = 6.63, p < 0.0001$, Low $\mathcal{Q}_{\text{Same}}$ Group: $T(29) = 2.25$, $p < 0.05$]. The difference in correlation, $\Delta\mathcal{Q}$ was greater for the high-$\mathcal{Q}_{\text{Same}}$ group [$T(58) = -3.43$, $p < .01$]. This is in the opposite direction than expected, thus arguing against the notion that the large value of $\Delta\mathcal{Q}$ is an artifact of low test/re-test correlation.

The most notable difference in experimental design being the PAL and probed SL experiments is the perfect serial recall criterion in the SL experiment compared to imperfect levels of cued recall learning in the PAL experiment. Thus, perhaps $\Delta\mathcal{Q}$ increases as the materials are overlearned. Similar to the previous comparison, I median-split participants based on their median number of trials to criterion (TTC) in the initial serial recall phase of the probed SL experiment (median TTC = 5 trials). As can be seen in Fig. 1b, $\Delta\mathcal{Q}$ remains large for both groups of participants. $\mathcal{Q}_{\text{Same}}$ was significantly different from $\mathcal{Q}_{\text{FB}}$ for both groups [Low-TTC group: $T(28) = 2.97, p < 0.01$, High-TTC group: $T(30) = 5.56$, $p < 0.0001$]. $\Delta\mathcal{Q}$ did not differ between groups [$T(58) = -1.48, p > 0.1$]. Thus, the low correlation between forward and backward probes replicates across participants with, on average, different numbers of study trials.

I also asked whether degree of learning could be modulating the forward–backward correlation across lists by median-splitting lists within participants. Fig. 1c shows that the $\Delta\mathcal{Q}$ remains large for both rapidly and slowly learned lists. Some participants had to be excluded from this analysis if they had no range of TTC values. $\mathcal{Q}_{\text{Same}}$ was significantly different from $\mathcal{Q}_{\text{FB}}$ for both low- and high-TTC lists [Low-TTC lists: $T(49) = 3.68, p < 0.001$, High-TTC lists: $T(49) = 2.16$, $p < 0.05$]. $\Delta\mathcal{Q}$ was greater for low-TTC lists than for high-TTC lists [$T(49) = -2.08, p < 0.05$]. This suggests that degree of learning, if anything, *reduces* our observed effect.

Another way of asking whether degree of learning could be modulating the forward–backward correlation is by comparing serial positions *within lists*. I did this by analyzing separately probes of early serial positions (both probe and target items in serial positions 3–10), which tend to be learned early in study-test trials versus probes late serial positions (remaining positions) which tend to be learned in later trials. Some participants had to be excluded from this analysis if there was insufficient data to compute one of the three $\mathcal{Q}$ values. Fig. 1d shows that $\Delta\mathcal{Q}$ is significant both for early and late positions [early: $T(59) = 3.26, p < 0.01$, late: $T(59) = 3.95$, $p < 0.001$] and $\Delta\mathcal{Q}$ did not differ between early and late positions [$T(59) = 0.52, p > 0.5$].

In sum, despite the fact that the PAL and SL data come from different experiments, the extremely high correlation between forward and backward probes of paired associates contrasts with the substantially low forward–backward correlation found in probed serial lists. The correlation is not a simple consequence of lower test/re-test correlation in probed SL and replicates across degrees of learning across participants, lists and serial positions. While additional follow-up experiments are warranted to further characterize the boundary conditions of this dissociation, it stands as an empirical finding to be addressed theoretically. The remainder of this article will examine whether dissociations of this nature can be reconciled with models that assume common mechanisms for PAL and SL.

2. The isolation hypothesis

The near-perfect correlation in PAL simply means that forward and backward probed recall are influenced by common sources of variability. Whatever processes account for the majority of the variance must influence forward and backward recall in the same way. The convolution/correlation formalism (Borsellino and Poggio, 1971) naturally satisfies this condition (Murdock, 1979), and the Matrix Model formalism may be constructed to satisfy this (Rizzuto and Kahana, 2001). For continuity with this latter work, I will follow the Matrix Model formalism. Dissociations in performance could suggest that PAL and SL obey different basic principles. However, here I attempt to treat both paradigms within a common framework, using a single, continuous-valued parameter to move a memory model from PAL to SL behavior.

I propose the following. Forward and backward probed recall have similar dynamics in PAL as in SL. What differentiates PAL from SL is that in PAL, paired items at studied positions $\{x, x+1\}$, are relatively isolated from other studied items at positions $y \neq x, x+1$ (Woodworth, 1915; Murdock and Franklin, 1984). Thus, what dominates the correlation between forward and backward probe accuracy in PAL is variability in storage operations involving x and $x+1$, which must be highly correlated to account for the reliable data showing high measured correlations in PAL. In a chaining formalism this corresponds to the association strength between item x and item $x+1$. In a positional coding formalism, this corresponds to variability in encoding of item–position and position–item associations for items x and $x+1$. In contrast, in SL, probed recall is susceptible to competition from items that were nearby in study. Under appropriate conditions (which will become evident during exposition of the models), the degree of competition from particular items can differ for forward versus backward probe directions. If these direction-dependent terms are relatively independent, they will reduce the correlation. Next I elaborate isolation in an associative chaining model and then in a positional coding model.

Modelling strategy: The purpose of the model derivations is to test whether the Isolation Principle can operate in models, and in particular, whether the principle can function in both chaining and positional coding formalisms. Rather than develop a complete model of SL and PAL that can account for a broad range of existing data, I keep the models as simple as possible without being blatantly unrealistic. This will facilitate intuitions about how and why the Isolation Principle functions in both classes of models without being obscured by countless specific, debatable implementation choices. In particular, I consider it beyond the scope of this manuscript to attempt to account for serial position effects and learning over repeated presentations, although one hopes that the effects derived here are compatible with these sources of variability.

Further, the particular correlational measures of PAL and SL data considered here are not likely to select associative chaining over positional coding or vice versa. The aim in implementing Isolation in both positional and associative chaining models is not to select them against each other but rather, to demonstrate the generality of the notion of Isolation. By implementing this model-construction principle in a simple associative chaining model, might understand what the necessary conditions are for handling forward–backward probed recall correlation data within associative chaining frameworks. Likewise, by implementing the same type of principle in a simple positional coding model, we might find out how we have to constrain positional coding models to account for the correlational data presented here. While it would be ideal to make the models as comparable to one another as possible, it is not easy to do so; for example, the positional coding model necessarily requires more machinery (i.e., a representation of position) than the associative chaining models.

While I refer to more general formalisms with full vector representations, both the associative chaining and positional coding models presented here are derived as strength models. Importantly, both models both contain a single continuous-valued free parameter that implements the Isolation Principle, allowing the model to move from the perfectly isolated PAL regime to the completely unisolated SL regime.

The chief way in which the models differ is that the associative chaining model learns direct item–item associations. In contrast, the positional coding model learns no direct item–item associations; all item–item interactions must be mediated by the positional code via item–position and position–item associations. While it may be controversial that in this framework, even PAL involves no item–item associations, this is necessary to treat PAL and SL within the same framework. The notion of using positional coding to unify PAL and SL is quite vulnerable to empirical evidence if it could be demonstrated that PAL necessarily relies on direct, unmediated item–item associations.

For each model I first analytically derive continuous correlations between the outcomes of forward and backward probe operations just prior to response thresholding. Then, I solve for the discrete, Yule's $\mathcal{Q}$ values using simulations that include response-thresholding.

2.1. Example 1: isolation in an associative chaining model

Consider a Hebbian matrix model of associative chaining (e.g., McNaughton and Morris, 1987; Humphreys, Bain, & Pike, 1989; Humphreys, Pike, Bain, & Tehan, 1989; Rizzuto and Kahana, 2001). The model shall store a list of L words, indexed by l. I begin with a general storage equation to which I shall later add constraints.

Assumptions:

(1) *Item representation*: The first assumption is that items are represented as vectors with high dimensionality. I will follow the convention that vectors are set in bold face and assumed to be column vectors, prime ($'$) denotes the transpose, and vector dimensions are indexed in parentheses whereas subscripts (e.g., k and l) index different vectors. Thus, an item, $\mathbf{f}_l$ (or $\mathbf{f}_k$) is an N_f-dimensional vector whose elements are independent, identically distributed (i.i.d.), Gaussian random variables with a mean of zero and a variance of $1/N_f$. I assume that the item representation vectors have high dimensionality so that I can simplify the subsequent derivations by taking the limit $N_f \to \infty$. This implies that $\mathbf{f}_k \cdot \mathbf{f}_l = \delta_{kl}$, where δ_{kl} is Kronecker's Delta (equal to 1 when $k = l$ and 0 otherwise). This can be viewed as a strength model. I nonetheless set up the formalism of the model using vector notation to emphasize that a realistic implementation of the model would need to be based on some finite-dimensional vector representation (e.g. Caplan, 2004). Insofar as this dimensionality is high, the present derivations will be relevant. If one were to reduce the dimensionality of the item representation, that would allow for item–confusion errors, which could differentially influence the probe versus target word depending on probe direction; if these effects are large, the correlation between forward and backward probes should decrease substantially, even for PAL.

(2) *Forward versus backward associations*: The second assumption regards the problem of target ambiguity in chaining models (Kahana and Caplan, 2002). That is, when probed, the model cannot distinguish the forward from the backward association. In serial recall, this can be overcome by response suppression (Lewandowsky and Murdock, 1989), but in the case of a single-item probe, there are no relevant items to suppress. This imposes a theoretical upper limit of 50% accuracy. To enable the model to overcome target ambiguity and, in principle, be able to perform better than 50% accuracy, I assume that the model stores forward and backward associations in separate memory matrices, W_+ and W_-,

respectively. that participants can selectively and accurately probe the correct memory matrix, W_+ or W_-, depending on the direction of the probe. One could easily relax this assumption without substantially changing the outcome of the present derivations. For example, instead of only probing the correct memory matrix, the model could probe with a weighted sum of the correct- and incorrect-direction matrix. This would restore some target ambiguity at study or at test, but would complicate the derivations while not adding substantially to our intuition about how isolation influences the forward/backward correlation.

(3) *Remote associations*: The third assumption is that, for simplicity, and to model the effects of interference, the model only stores associations between nearest-neighbor items (Lewandowsky and Murdock, 1989; Murdock and Franklin, 1984) and nearest-but-one neighbor (the closest "remote" associations) items. In the strict PAL regime (perfect isolation), these remote associations will be zero. However, if isolation is even slightly imperfect, the remote associations will be non-zero (albeit weak). The same applies to between-pair associations. Thus, in addition to modelling the strict PAL and SL regimes, I allow for intermediate regimes with moderate levels of isolation. These intermediate regimes could correspond to real-life or experimental situations in which participants may not follow strict PAL or SL strategies. Further, allowing intermediate regimes allows us to examine how model behavior moves between the strict regimes. Also, because I am not considering memory for item information here, I assume, for simplicity, that no autoassociations are stored in either PAL or SL.

(4) *Response screening*: The fourth assumption is that the model prevents itself from recalling the probe item which it is given. Rather than complicate the model by implementing this as a fallible process, I simply assume that the model can achieve this perfectly. This seems plausible in most probe paradigms, in which the probe remains visible on the screen until the response is made or the time allowed for responding expires.

(5) *Net retrieval strength*: The fifth assumption is that for each probe operation, every possible item has a retrieval strength. The model will produce the correct response if the retrieval strength of the target both exceeds a response threshold and exceeds the sum of the retrieval strengths over all incorrect items (excluding the probe item; see previous assumption). Thus, recall success depends on what I term *net strength*, defined as the difference between retrieval strength of the target and the sum of the retrieval strengths of all competing items. This net strength can be seen as an approximation of a probabilistic, competitive retrieval rule such as that suggested by Luce (1959). That the target strength must exceed the *sum* of the competitor strengths as well as a threshold can be thought of as collapsing together several types of response errors: (i) omissions due to all strengths being weak, (ii) omissions due to large levels of competition resulting in many items being sampled until a stopping limit is reached before the (strong) target item is sampled and (iii) intrusions wherein an incorrect item's strength alone exceeds the target strength as well as a response threshold.

Study: The storage equation involves two symmetric operations:

$$W_+ = \sum_{l=1}^{L-1} \sum_{k=l+1}^{L} \Gamma_{lk} \mathbf{f}_k \mathbf{f}'_l,$$

$$W_- = \sum_{l=1}^{L-1} \sum_{k=l+1}^{L} \Gamma_{kl} \mathbf{f}_l \mathbf{f}'_k.$$

Encoded associative strength is controlled by the variables Γ_{lk}. To emphasize the pattern of isolation, I write Γ as the product of two variables

$$\Gamma_{lk} = S_{lk}\gamma_{lk}.$$

The γ_{lk} is a randomly drawn encoding strength,[3] and each strength is scaled by a constant S_{lk} that depends on $l-k$. S_{lk} will control the degree of isolation, and are further specified below. γ_{lk} is an independent random variable such that $\mathrm{E}[\gamma_{lk}] = 1$ and $\mathrm{var}[\gamma_{lk}] = \sigma^2$. Thus, $\mathrm{E}[\Gamma_{lk}] = S_{lk}$ and $\mathrm{var}[\Gamma_{lk}] = S_{lk}^2\sigma^2$. To make the basic storage mechanism embody associative symmetry, I assume that forward and backward association terms are stored in a perfectly correlated manner; i.e.,

$$\gamma_{lk} \equiv \gamma_{kl}. \tag{1}$$

The pattern of isolation is captured by S_{lk}, which are constant scalars that weight terms differently. By constraining the values of S_{lk}, one can mimic various

[3]This is a no-forgetting chaining model. Lewandowsky and Murdock (1989) implemented a chaining model with forgetting; upon each encoding operation, a forgetting parameter, α, scaled down the strength of the previously learned associative information, although the fit values of α are typically just under 1 in order to explain the asymptotic portion of the serial position curve (Murdock and Hockley, 1989). Including a sub-unity value of α in the chaining model discussed here should provide a better fit to serial recall data. However, doing so would complicate the derivations, without undermining the key insights. In particular, any parameters that produce serial position variability would increase correlations between forward and backward probes of adjacent list items. Nonetheless, to explain dissociations between cued recall of pairs versus lists, it is still necessary to analyze of isolation and the effects of within-list interference. The present model is also an unlimited capacity model; because we are not trying to account for item–memory effects at this time, there is no need to include the additional machinery necessary to account for tradeoffs between learning and forgetting of item versus associative information (Hockley, 1992; Murdock, 1993).

standard models. For this model,

$$S_{lk} = \begin{cases} S_{\mathrm{within}} & |l-k| = 1 \text{ within pair}, \\ S_{\mathrm{between}} & |l-k| = 1 \text{ between pair}, \\ S_{\mathrm{remote}} & |l-k| = 2, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where $S_{\mathrm{remote}} = S_{\mathrm{r}} \times S_{\mathrm{between}}$,

$S_{\mathrm{r}} \leqslant 1$ and $S_{\mathrm{between}} \leqslant S_{\mathrm{within}}$,

where S_{within}, S_{between} and S_{r} are constant scalars. The reason for defining S_{remote} as a product of S_{r} with S_{between} is to ensure that if S_{between} is small, the remote associations will necessarily be small; thus, the remote associations are subject to the same effects of isolation, over and above the remote association strength simply being less than adjacent association strength. The degree of isolation is defined as

$$I = 1 - \frac{S_{\mathrm{between}}}{S_{\mathrm{within}}}. \tag{3}$$

In the SL limit, $S_{\mathrm{between}} = S_{\mathrm{within}}$ and $I = 0$, whereas in the PAL limit, $S_{\mathrm{between}} = 0$, $S_{\mathrm{within}} > 0$ and $I = 1$. Of crucial importance is the distinction between *within*- and *between*-pair association strengths. This distinction is made in PAL but vanishes in the SL limit. The PAL conditions are an implementation of *associative chaining isolation*, where the strength of within-pair associations is much greater than the strength of between-pair associations.

Probed recall: Consider a specific list transition, $\{x, x+1\}$. The probe operation begins by multiplying a probe item vector onto the appropriate memory matrix, W_+ or W_- to obtain a retrieved item vector, $\mathbf{f}_{\mathrm{r}}$:

$$\text{Forward: } \mathbf{f}_{\mathrm{r}} = W_+ \mathbf{f}_x,$$
$$\text{Backward: } \mathbf{f}_{\mathrm{r}} = W_- \mathbf{f}_{x+1}.$$

The model computes a retrieval strength, a_l, for each list item l, where similarity between the retrieved vector, $\mathbf{f}_{\mathrm{r}}$, and all list items, where

$$a_l = \mathbf{f}_{\mathrm{r}} \cdot \mathbf{f}_l.$$

Accuracy depends on whether the target strength, a_{x+1} (or a_x), can exceed the strengths of the distractor items, a_p, and a response threshold, θ. For the forward probe, the strengths a_{x+1} and a_p are

$$a_{x+1} = (W_+ \mathbf{f}_x) \cdot \mathbf{f}_{x+1} = \sum_{l=1}^{L-1} \sum_{k=l+1}^{L} S_{lk} \gamma_{lk} (\mathbf{f}_l \cdot \mathbf{f}_x)(\mathbf{f}_k \cdot \mathbf{f}_{x+1}),$$

$$a_p = (W_+ \mathbf{f}_x) \cdot \mathbf{f}_p = \sum_{l=1}^{L-1} \sum_{k=l+1}^{L} S_{lk} \gamma_{lk} (\mathbf{f}_l \cdot \mathbf{f}_x)(\mathbf{f}_k \cdot \mathbf{f}_p),$$

$$p \neq x, x+1.$$

Intrusions are possible but I only derive expressions relevant to recall probability for the correct target item, classifying all other types of outcomes as incorrect. Probing in the backward direction, the target and distractor strengths, a_x and a_q, are

$$a_x = \sum_{l=1}^{L-1} \sum_{k=l+1}^{L} S_{lk} \gamma_{lk} (\mathbf{f}_l \cdot \mathbf{f}_{x+1})(\mathbf{f}_k \cdot \mathbf{f}_x),$$

$$a_q = \sum_{l=1}^{L-1} \sum_{k=l+1}^{L} S_{kl} \gamma_{kl} (\mathbf{f}_k \cdot \mathbf{f}_{x+1})(\mathbf{f}_l \cdot \mathbf{f}_q),$$

$$q \neq x, x+1.$$

Define *net strength* (denoted ξ_{F} for forward and ξ_{B} for backward probes) as the strength of the target item minus the sum of the strengths of all distractors. Then, the target item will be correctly recalled if its ξ_{F} (or ξ_{B}) exceeds the response threshold, θ. Otherwise, either a distractor item would be recalled or an omission would be made. Recall that ξ_{F} and ξ_{B} rely on underlying random variables. Thus, the probability of correct recall in the forward and backward directions are, respectively:

$$\text{Forward: } P(F) = P(\xi_{\mathrm{F}} > \theta),$$
$$\xi_{\mathrm{F}} = a_{x+1} - \sum_{p \neq x, x+1} a_p,$$
$$\text{Backward: } P(B) = P(\xi_{\mathrm{B}} > \theta),$$
$$\xi_{\mathrm{B}} = a_x - \sum_{q \neq x, x+1} a_q.$$

For the model considered here, the only non-zero distractor terms are those for the remote associations $p = x+2$ (forward probes) and $q = x-1$ (backward probes). The net strengths can be rewritten:

$$\xi_{\mathrm{F}} = a_{x+1} - a_{x+2} = S_{x,x+1}\gamma_{x,x+1} - S_{x,x+2}\gamma_{x,x+2}, \tag{4}$$

$$\xi_{\mathrm{B}} = a_x - a_{x-1} = S_{x+1,x}\gamma_{x+1,x} - S_{x+1,x-1}\gamma_{x+1,x-1}. \tag{5}$$

Analytical solution for correlation in net strength: One can anticipate the result at this stage by observing that a_x and a_{x+1} are identical (Eqs. (4) and (5)), because $\gamma_{x,x+1} = \gamma_{x+1,x}$ (Eq. (1)). However, for the competing terms, this depends on the degree of isolation. In the case of PAL, $S_{x,x+2} = S_{x+1,x-1} = 0$, which means that ξ_{F} and ξ_{B} will be dominated by the perfectly correlated a_{x+1} and a_x. In the case of SL, however, $a_{x,x+2} \neq a_{x+1,x-1}$, and with the present constraints, are independent. Thus, item competition will tend to counteract the high correlation induced by the retrieval strength of the target. The less the degree of isolation, the greater $S_{x,x+2}$ and $S_{x+1,x-1}$ will be, and the lower the correlation between ξ_{F} and ξ_{B}, and thus between forward and backward recall.

First I will obtain a closed-form solution for the continuous correlation between ξ_{F} and ξ_{B}. However, the inclusion of a response threshold introduces an important nonlinearity into the expressions for retrieval probability. I will examine the effect of the threshold by computing Yule's $\mathcal{Q}$ in numerical simulations. To solve for the continuous correlation I use

$$\mathrm{cov}[\gamma_{x,x+1}, \gamma_{x+1,x}] = 1,$$

$$\begin{aligned}\mathrm{cov}[\gamma_{x,x+1}, \gamma_{x,x+2}] &= \mathrm{cov}[\gamma_{x,x+1}, \gamma_{x+1,x-1}] \\ &= \mathrm{cov}[\gamma_{x,x+2}, \gamma_{x+1,x-1}] = 0.\end{aligned}$$

The continuous correlation, ρ_{FB}, between ξ_{F} and ξ_{B} is

$$\rho_{\mathrm{FB}} = \frac{\mathrm{cov}[\xi_{\mathrm{F}}, \xi_{\mathrm{B}}]}{\sqrt{\mathrm{var}[\xi_{\mathrm{F}}]\mathrm{var}[\xi_{\mathrm{B}}]}}. \quad (6)$$

Given that

$$\begin{aligned}\mathrm{cov}[\xi_{\mathrm{F}}, \xi_{\mathrm{B}}] &= \mathrm{cov}[a_{x+1} - a_{x+2}, a_x - a_{x-1}] \\ &= \sigma^2 S_{x,x+1} S_{x+1,x},\end{aligned}$$

$$\mathrm{var}[\xi_{\mathrm{F}}] = \mathrm{var}[a_{x+1} - a_{x+2}] = \sigma^2(S^2_{x,x+1} + S^2_{x,x+2}),$$

$$\mathrm{var}[\xi_{\mathrm{B}}] = \mathrm{var}[a_x - a_{x-1}] = \sigma^2(S^2_{x+1,x} + S^2_{x+1,x-1}).$$

Eq. (6) becomes

$$\rho_{\mathrm{FB}} = \frac{S_{x,x+1} S_{x+1,x}}{\sqrt{(S^2_{x,x+1} + S^2_{x,x+2})(S^2_{x+1,x} + S^2_{x+1,x-1})}}. \quad (7)$$

Or, substituting the expressions for S_{lk} (Eq. (3)),

$$\rho_{\mathrm{FB}} = \frac{S^2_{\mathrm{within}}}{(S^2_{\mathrm{within}} + S^2_{\mathrm{between}} S^2_{\mathrm{r}})}.$$

Even if we were to add an asymmetry, α, such that

$$\alpha = \frac{S_{y,x}}{S_{x,y}} \quad \text{where } x > y \quad (8)$$

the α factor is present both in the numerator and denominator, canceling in both the PAL and the SL regimes. This way of implementing asymmetry is one example that shows that the measurement of (a)symmetry of mean performance does not necessarily tell us what the correlation will be (and vice versa).

As an aside, one can easily follow the effect of added remote associations by simply making the following replacements:

$$\mathrm{E}[\xi_{\mathrm{F}}] = S_{x,x+1} - \sum_{y=x+2}^{L} S_{x,y}$$

$$\mathrm{var}[\xi_{\mathrm{F}}] = \sigma^2 \sum_{y=x+1}^{L} S^2_{x,y}, \quad (9)$$

$$\mathrm{E}[\xi_{\mathrm{B}}] = S_{x+1,x} - \sum_{y=1}^{x-2} S_{x+1,y}$$

$$\mathrm{var}[\xi_{\mathrm{B}}] = \sigma^2 \sum_{y=1}^{x} S^2_{x+1,y}. \quad (10)$$

In computing ρ_{FB}, the numerator remains the same, but the denominator accrues extra terms due to the additional remote associations. Thus, Eq. (7) becomes

$$\rho_{\mathrm{FB}} = \frac{S_{x,x+1} S_{x+1,x}}{\sqrt{(S^2_{x,x+1} + \sum_{y=x+2}^{L} S^2_{x,y})(S^2_{x+1,x} + \sum_{y=1}^{L-1} S^2_{x+1,y})}}. \quad (11)$$

The effect is that adding increasing numbers of competing associations drives down the correlation. Again, in the PAL limit, the interfering terms approach zero, making the correlation approach unity. In contrast, in SL, those terms are non-zero, and the stronger they are, the lower the correlation becomes. Remote associations (or any uncorrelated associations), thus, serve to further accentuate the difference between PAL and SL regimes.

Yule's $\mathcal{Q}$ solution by simulation: Next I simulate the model with threshold by generating pseudo-random Gaussian-distributed values for the γ variables in the expressions for the net strengths (Eqs. (4) and (5)). Responses were considered correct if the net strength exceeded the response threshold, θ; i.e., $\xi_{\mathrm{F}} > \theta$ or $\xi_{\mathrm{B}} > \theta$ for forward and backward probes, respectively. Yule's $\mathcal{Q}$ between forward and backward probes, $\mathcal{Q}_{\mathrm{FB}}$, was computed from the contingency table assembled from response correctness across 10^6 simulated item-pairs $\{x, x+1\}$. Simulations were run for values of I (parameterizing isolation) ranging from 0.05 to 1 in steps of 0.05. Note that I (Eq. (3)) is, more specifically,

$$I = 1 - \frac{S_{x+1,x+2}}{S_{x,x+1}} \equiv 1 - \frac{S_{x,x-1}}{S_{x+1,x}}.$$

Fig. 2 shows the dependence of the correlations on I as well as three model parameters of interest: θ, the coefficient of variation of encoding strength, CV_γ, and the degree of asymmetry, α (Eq. (8)).The plots illustrate the case where $S_{\mathrm{remote}} = 0.9 S_{\mathrm{within}}$. If S_{remote} is lowered (not shown), this would tend to reduce the sources of uncorrelated interference, driving up the correlation; increasing S_{remote} has the reverse effect. The dashed lines plot the Pearson correlation, ρ_{FB} between forward and backward net strengths, illustrating that as degree of isolation increases, ρ_{FB} increases. All plots of ρ_{FB} overlap, reflecting the fact that θ does not enter into the analytic solution and CV_γ and α cancel out and thus are absent from the final expression (Eq. (7)).

The thresholded model similarly shows that $\mathcal{Q}_{\mathrm{FB}}$ as I increases, reaching unity with perfect isolation. However, Yule's $\mathcal{Q}$ does not, for reasonable parameter values, take on as low values as does ρ_{FB}. As can be seen in Fig. 2a, as θ increases (think of a very conservative participant who only very high net-strength items), the entire curve scales upward. Thus, the thresholded model gives correlations between response accuracy that are inflated relative to the correlations between the underlying retrieval strengths.

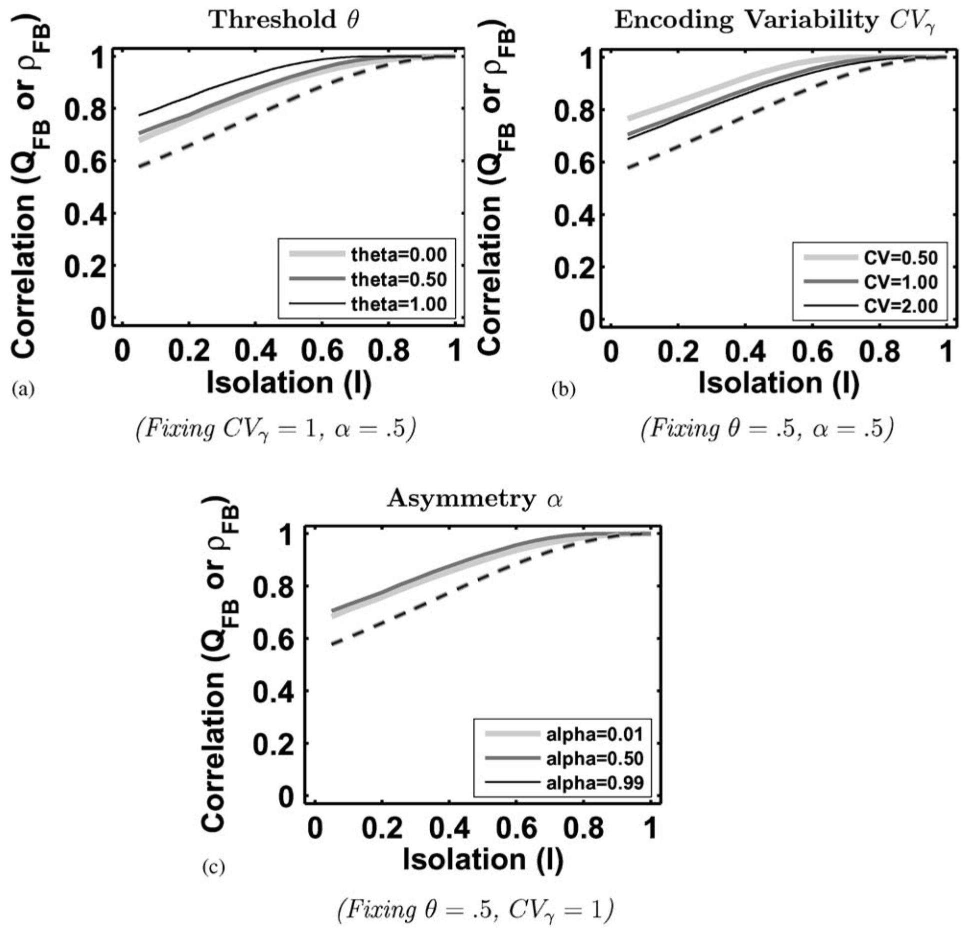

Fig. 2. Effect of isolation and dependence on parameters of the associative chaining model. In all panels, the relative strength of remote associations, S_{remote}, is fixed at $0.9S_{\text{within}}$. Dashed lines plot the closed-form solutions for the Pearson correlation (ρ_{FB}) between net strengths for forward and backward probes as a function of the isolation parameter, I (Eq. (7)). Large values of I represent maximal isolation, in which competing association strengths are near zero; small values of I represent minimal isolation, in which competing association strengths are of comparable to target association strengths. Solid lines plot the results of the numerical simulations (see text for details), illustrating how $\mathcal{Q}_{\text{FB}}$ varies as a function of the I as well as three parameters of interest: the response threshold, θ (a, holding constant CV_γ and α), the coefficient of variation of encoding strength, CV_γ (b, holding constant θ and α) and α, where $\alpha = \frac{S(t_x,t_y)}{S(t_y,t_x)}, x<y$ (c, holding constant θ and CV_γ).

Encoding strength variability, CV_γ, has a small effect on $\mathcal{Q}_{\text{FB}}$, primarily at low levels of isolation (Fig. 2b). In other words, if encoding is very reliable, then the probe–target association will consistently exceed the distracting association strengths, thus having a similar effect as increased isolation.

Asymmetry (Fig. 2c) has no effect over a broad range of α values, in the thresholded model as well as in the unthresholded analytical model. This confirms the mathematical fact that the symmetric mean performance does not necessarily reflect correlated forward and backward probes and vice versa.

Even at the lowest isolation level, ρ_{FB} and $\mathcal{Q}_{\text{FB}}$ are substantially positive. This is because in the present formalism, forward and backward probes rely on common, correlated sources of variance. Thus, the Isolation Principle does not generally imply zero correlations, but rather, differences in correlation between experimental conditions.

2.1.1. Discussion

Paralleling existing chaining models of SL, I showed that this approach indeed produces the desired dissociation in correlations between PAL and SL. The difference between PAL and SL was modulated by a single parameter that controlled the strength of between-pair nearest-neighbor association terms. The success of the derivations relied on the following properties of the model.

First, forward and backward associations were stored in the same manner at study and were probed in the same way at test.

Second, the variability terms picked up by forward and backward probes were the same. This was because the forward and backward association strength terms were perfectly correlated. In convolution/correlation models, this is the only possible case (e.g., Lewandowsky and Murdock, 1989); forward and backward association terms are in fact the same term. In matrix implementations, one can make them different but even low levels of independence begin to lose the near-perfect correlation in PAL (Kahana, 2002).

Third, the dissociation in the correlation for PAL and SL performance was a consequence of retrieval processes, but relied on conditions at study (average relative strength of between- to within-pair associations). Forward and backward probes used the same processes but were differently susceptible to interference from neighboring items depending on isolation established at study.

Finally, consider the Within-List Control correlation. In this chaining model, association strengths are independent across a list. To match the considerable levels of this control correlation seen in the data (Table 2), one could introduce such correlations, for example, by introducing list-to-list variability in $E[\gamma_{lk}]$.

While simple associative chaining models have been challenged by empirical findings (e.g., Baddeley, 1968; Henson et al., 1996; Wickelgren, 1966) it is not clear that more complex chaining models (e.g., with remote associations) could not overcome such challenges. Furthermore, the ability of participants to perform at high levels of accuracy on circular lists, in which positional information is disrupted (Addis and Kahana, in preparation) suggests that chaining-like information may be used in circumstances in which positional information is less diagnostic, although relative-order models could potentially accommodate certain features of circular list learning. It may be that subjects use a variety of cues to learn and retrieve serial lists, some of them positional-like and some of them chaining-like (e.g. Giurintano, 1973; Maisto and Ward, 1976; Woodward and Murdock, 1968; Young, 1968). Thus, it seems more useful to demonstrate the generality of the Isolation Principle than to suggest that it is highly model-dependent.

2.2. Example 2: isolation in a positional coding model

A positional coding model (e.g. Brown et al., 2000; Burgess and Hitch, 1999; Henson, 1998; Lee and Estes, 1977; Lewandowsky and Farrell, 2000; Page and Norris, 1998) learns to link up a set of positions $\mathbf{t}_k$, indexed by k, with a separate set of items $\mathbf{f}_l$, indexed by l, where $\mathbf{f}_l$ and $\mathbf{t}_k$ are vectors with dimensionalities N_f and N_t, respectively.

Assumptions:

(1) *Item and position representations*: While items and positions might be written as vectors, I will not make direct use of any particular vector representation, reducing to a strength model. Thus, I drop the bold, vector typesetting and write items and positional codes as scalars, f_l and t_k, respectively.

(2) *Position–item associations*: The second assumption is an extension to positional coding models to enable the model to perform cued recall with sequential probes, probing with one item for another item. To this effect, I make the simplest possible extension to this class of model—that is, that as the model learns associations from t_k to f_l, it simultaneously learns associations from f_l to t_k (cf. Brown et al., 2000). Upon a sequential probe, the model retrieves the position associated with the cue item, and moves to the desired relative position and probes with this new positional code to retrieve a target item.

(3) *PAL in a positional coding model*: The third assumption extends positional coding models to paired associates learning paradigms. My approach has some similarity to that taken by Mensink and Raaijmakers (1988), who use a contextual variable that is, in some ways, an analogue of positional coding. This is done by assuming that, in the PAL regime, paired items are assigned to very similar positions relative to the positions of other list items. This positional coding model differs critically from the associative chaining model in that it learns only item–position and position–item associations but no direct item–item associations.

(4) *Positional similarity*: The fourth assumption is that when the model manipulates positional codes, they may be confused with one another according to a similarity structure, where $S(t_l, t_k)$ denotes the similarity between two positions, t_l and t_k. I always assume that $S(t_l, t_l) = 1$. The only way in which PAL and SL shall differ is in the similarity structure of the positional vectors, t_k across the list (see Fig. 3 for a schematic representation). Without making specific assumptions about the nature of the representations of positional codes, I assume that the similarity between positional codes is:

$$S(t_l, t_k) = \begin{cases} e^{-(1-\varDelta)/\tau} & |l-k| = 1 \text{ within pair,} \\ e^{-\varDelta/\tau} & |l-k| = 1 \text{ between pair,} \\ \prod_{m=l}^{k-1} S(t_m, t_{m+1}) & |l-k| > 1, \end{cases} \tag{12}$$

where $0.5 \leqslant \varDelta \leqslant 1$.

Here, τ sets the width of the positional similarity function; the larger τ, the more dissimilar are nearby list items' positional codes. However, for any given comparison between PAL and SL (as well as intermediate regimes), I hold τ constant. τ is thus assumed to be fixed between paradigms, so I am relying on the difference between within-pair positional similarity and between-pair similarity to account for behavioral dissociations. The scalar, $\varDelta$, relative to τ, parameterizes isolation from a maximally isolated model ($\varDelta = 1$), in which similarity of positional codes from one pair to another is nearly zero, to a minimally isolated model ($\varDelta = 0.5$), in which there is no difference between

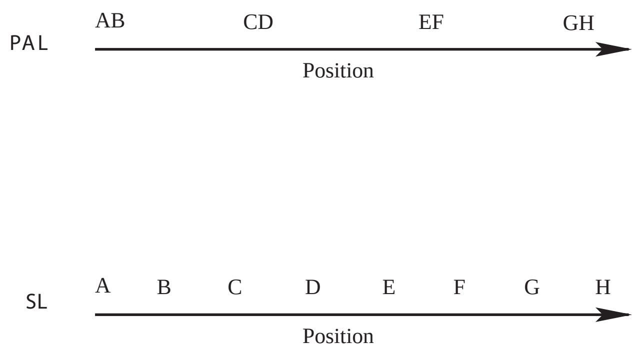

Fig. 3. Positional isolation. In a positional coding model, the Isolation Principle is embodied by relative spacing of the positional codes. In paired associates learning (PAL), the positional codes for two items within a pair are spaced close to each other, but far from other pairs. In serial learning (SL), positional spacing is variable and there is no distinction between within- and between-pair transitions.

within- and between-pair similarity. For the SL case, Eq. (12) reduces to

$$S(t_l, t_k) = e^{-|l-k|/2\tau}.$$

In SL, similarity decreases monotonically with increasing lag. This is the effect achieved in the Perturbation Model (Lee and Estes, 1977) as well as in certain contextual models (e.g. Howard and Kahana, 2002; Mensink and Raaijmakers, 1988). In PAL, positional similarity is very high within a pair but very low between pairs. This is equivalent to the statement that the A and B items within a pair are associated with (almost) identical positions. In this way, I implement *positional isolation*.

(5) *Shifting positions*: The fifth assumption is that the model can move perfectly from one positional code, t_l, to another positional some relative distance away, $t_l + T$, where T is an integer. This simplification (cf. Brown et al., 2000) keeps the model tractable, but remains to be tested.

(6) *Response screening*: As with the associative chaining model, this model does not recall the cue item that it is given, assumed to be an infallible process.

(7) *Net retrieval strength*: This assumption parallels that for the associative chaining model: For each probe operation, every possible item has a retrieval strength. Recall success depends on *net strength*, the difference between retrieval strength of the target and the sum of the retrieval strengths of all competing items.

Study: The storage equations for a list of L items (or $L/2$ pairs) are

$$\mathbf{w}_{ft}(f_l) = \gamma_l,$$
$$\mathbf{w}_{tf}(t_l) = \gamma_l,$$
where $1 \leqslant l \leqslant L$.

If N_{pool} is the number of items in the word pool then $\mathbf{w}_{ft}$ is a N_{pool}-dimensional vector that accumulates the (scalar) heteroassociative strengths from positions t_l to items f_l and $\mathbf{w}_{tf}$ is a L-dimensional vector that accumulates (scalar) heteroassociative strengths from items f_l to positions t_l. The γ_l are independent random scalar variables such that $\mathrm{E}[\gamma_l] = 1$ and $\mathrm{var}[\gamma_l] = \sigma^2$. While I use the same γ_l for both storage equations, $\mathrm{cov}[\gamma_l, \gamma_k] = 0$, $k \neq l$. That is, the storage strength for a given list item is independent of the storage strength for all other items. For the present derivations, all other associations are set to zero.

Probed recall: Consider the pair $\{x, x+1\}$. Probing in the forward direction, I follow the retrieval strength of the correct target and distractor items:

(1) Probe with the item, f_x. The "correct" position, t_x is retrieved with strength $\mathbf{w}_{ft}(f_x) = \gamma_x$.
(2) "Roll" the positional code ahead from $t_x \to t_{x+1}$.
(3) Probe with t_{x+1} and retrieve the correct item f_{x+1} with strength $\gamma_x \mathbf{w}_{tf}(t_{x+1}) = \gamma_x \gamma_{x+1}$. Other items (distractors) are retrieved with strength $S(t_{x+1}, t_y)\gamma_x\gamma_y$, for $y \neq x, x+1$.Recall that the model is prevented from recalling the probe item, f_x.

Note that I am assuming that even if the position of the probe item is not recalled, the (noisy) outcome of Step 1 is nonetheless passed through Steps 2 and 3 to possibly retrieve the target item (e.g. Lewandowsky and Murdock, 1989). The retrieval strengths are the products of the first and third steps.

For the backward probe, consider the same pair of items, $\{x, x+1\}$:

(1) Probe with the item, f_{x+1}. The "correct" position, t_{x+1} is retrieved with strength $\mathbf{w}_{ft}(f_{x+1}) = \gamma_{x+1}$.
(2) "Roll" the positional code *back* from $t_{x+1} \to t_x$.
(3) Probe with t_x and retrieve the correct item f_x with strength $\gamma_{x+1}\mathbf{w}_{tf}(t_x) = \gamma_x\gamma_{x+1}$. Other items (distractors) are retrieved with strength $S(t_x, t_y)\gamma_{x+1}\gamma_y$, for $y \neq x, x+1$. The model is prevented from recalling the probe item, f_{x+1}.

As in the associative chaining model, probability of recall is equal to the probability that the *net strength*, forward: ξ_F or backward: ξ_B (strength of the target minus the summed strength of the distractors), exceeds a response threshold, θ. These are:

$$\xi_F = \gamma_x\gamma_{x+1} - \sum_{y \neq x, x+1} S(t_{x+1}, t_y)\gamma_x\gamma_y, \tag{13}$$

$$\xi_B = \gamma_x\gamma_{x+1} - \sum_{y \neq x, x+1} S(t_x, t_y)\gamma_{x+1}\gamma_y. \tag{14}$$

Analytical solution for correlation in net strength: I derive the continuous correlation $\rho_{FB} = \mathrm{cov}[\xi_F, \xi_B]/\sqrt{\mathrm{var}[\xi_F]\mathrm{var}[\xi_B]}$, for $L = \infty$ (an infinite number of interfering items) in Appendix. The expressions for variance and covariance are in Eqs. (19) and (21), respectively.

Yule's $\mathcal{Q}$ solution by simulation: Next I simulate the model with threshold as done previously for the associative chaining model, generating pseudo-random Gaussian-distributed values for the γ variables in Eqs. (13) and (14). Responses are considered correct if the net strength exceeds a threshold, θ: $\xi_F > \theta$ or $\xi_B > \theta$ for forward and backward probes, respectively. Terms involving $S(t_x, t_y)$ were included only for $S(t_x, t_y) > 0.001$. Yule's $\mathcal{Q}$ is computed from the contingency table assembled from response correctness across 10^6 simulated item-pairs $\{x, x+1\}$ and for varying degrees of isolation, parameterized by I ranging from 0.05 to 1 in steps of 0.05, where

$$I = 1 - \frac{e^{-\Delta/\tau}}{e^{-(1-\Delta)/\tau}}.$$

Fig. 4 shows the dependence of the correlations on I.Note that with the constraint that the positional distance between item x and item $x+1$ must be equal to 1, not all levels of isolation are possible for a given value of τ.

Similar to the associative chaining model, the model with response threshold gives correlations in accuracy that are inflated relative to the correlations between the underlying retrieval strengths (except for the case where θ is very low; in this case, the threshold serves only to

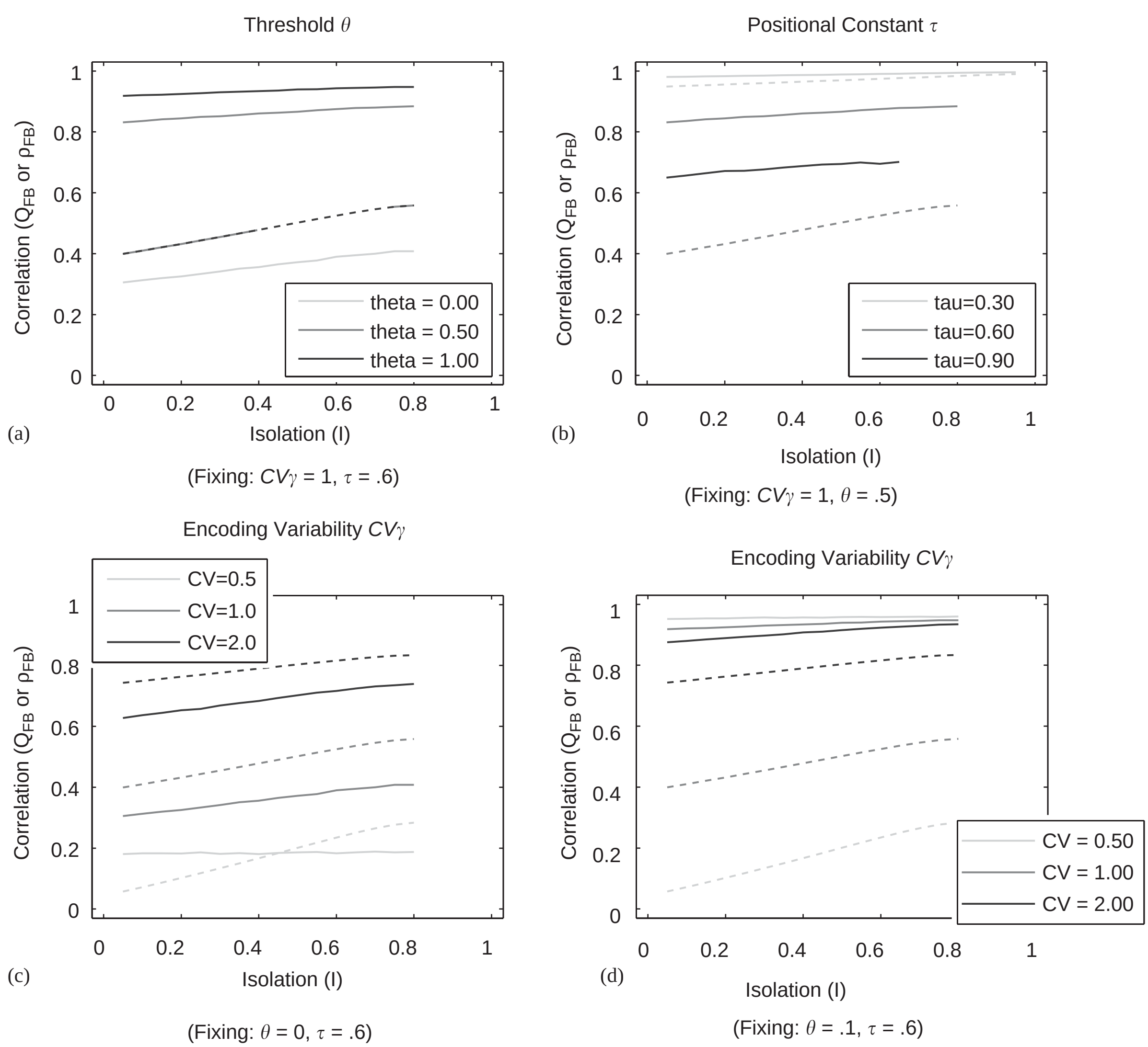

Fig. 4. Effect of isolation and dependence on parameters of the positional coding model. Dashed lines plot the closed-form solutions for the Pearson correlation (ρ_{FB}) between net strengths for forward and backward probes as a function of the isolation, I. Large values of I represent maximal isolation, in which competing association strengths are near zero; small values of I represent minimal isolation, in which competing association strengths are of comparable to target association strengths. Solid lines plot the results of the numerical simulations (see text for details), illustrating how Yule's $\mathcal{Q}$ varies as a function of the I as well as three parameters of interest: the response threshold, θ (a, holding constant CV_γ and τ), τ (b, holding constant CV_γ and θ) and CV_γ (holding constant θ and τ) both for a low and a high value of τ (c and d, respectively).

exclude negative values of net strength, perhaps removing some source of correlation). Nonetheless, the thresholded simulation shows predominantly similar dependence on parameters as the unthresholded, analytic solutions. Most pertinently, increased isolation leads to increased correlation. While for some parameter values, the dependence of the correlation on I is very slight, in no cases does the correlation decrease with increasing values of I.

Unlike the associative chaining model, competing list items provide both correlated and uncorrelated sources of variance to forward and backward probes. In particular, for forward and backward probes, the sources of interference are identical; they are simply weighted differently. This leads to richer dependence on model parameters and their interactions, which makes it difficult to intuit (without solving the model) what the character of dependence on τ, CV_γ (coefficient of variation of encoding strength γ_{lk}), I and θ will be. This characteristic also accounts for the weaker dependence on I compared to the chaining model. However, this does not reflect a fundamental difference between positional coding and associative chaining models. Indeed, the chaining model described above could be amended to make it more like the positional coding model by weakening the distinction between the forward and backward memory matrices, W_+ and W_- and including additional remote associations (Eqs. (9)–(11)).

Increased θ leads to increased correlation (Fig. 4a), similar to the associative chaining model; this increases the effective isolation of the model. Increased τ leads to reduced correlation (Fig. 4b), because the larger τ tends to increase the effective number of competing list items. Increased CV_γ tends to increase the correlation (Fig. 4c) because increased CV_γ offers more variability in probe and target strengths with which forward and backward probes can be correlated. The effect of varying CV_γ diminishes at high values of I. However, at very high values of θ, the dependence of $\mathcal{Q}_{\mathrm{FB}}$ on CV_γ can actually flip direction (observe that the rank-order of the different plots flips direction between panel d as compared to panel c of Fig. 4). This happens because at low levels of isolation, the decorrelating effect of the interfering list items is amplified by increased CV_γ. Again, the effect of CV_γ diminishes at high levels of I. Thus, CV_γ increases or decreases the correlation depending on how it interacts with other parameters and whether it introduces more effective variability into the probe and target terms or into the interference terms.

2.2.1. Discussion

By incorporating the Isolation Principle into a positional coding model, I was able to treat PAL and SL in a single framework. The model moves between PAL and SL by way of the similarity structure among the positional codes, which was implemented as a single parameter. This outcome depends on a number of conditions.

First, forward and backward probes functioned in the same way. This is analogous to properties of the associative chaining model, in which forward and backward associations were stored and retrieved in the same way.

Second, forward and backward probes picked up the same variability terms and, at least in the PAL regime, they combined multiplicatively. Since scalar multiplication is commutative, the total accumulated noise was independent of probe direction.

Third, the difference in correlation between forward and backward probes arises at test, and not at study but conditions at study (the relative positional spacing of items) are crucial to account for the observed pattern. Importantly, the strength of encoding of position-to-item and item-to-position terms were identical for a given list item. This was necessary; otherwise, forward and backward probes would have picked up different (and independent) noise terms, decorrelating them. Meanwhile, encoding strengths were independent across list items; this reduced the correlation for probed SL.

Finally, while the present model would produce zero correlations on the within-list control, inconsistent with Table 2, it would be straight-forward to add correlations among γ_l at different list positions that could fit such a non-zero baseline correlation.

3. General discussion

I first reported empirical evidence of a dissociation in the correlation between accuracy on forward and backward probes of SL as compared to PAL, presenting a challenge to models that treat PAL and SL using the same model processes and parameters. Then I showed that associative or positional isolation, along with constraints on noise terms within and between pairs, can modulate this correlation. In both classes of model, isolation was implemented as a single parameter that moved the model continuously from PAL to SL via a range of intermediate regimes.

In the case of an associative chaining model, I implemented the Isolation Principle by assuming that between-pair associations are stored more weakly than within-pair associations. Pairs were "associatively" isolated from the rest of the list. In the case of a positional coding model, I implemented the Isolation Principle by assuming that the positional code varies more between pairs than within pairs. Here pairs were isolated "positionally" from the rest of the list.

Both types of isolation resulted in highly correlated forward and backward recall in the PAL regime, compared with lower (non-zero and not perfect, albeit still rather high) correlations in the SL regime, with a single parameter continuously modulating this correlation. Crucially, in this framework, paired associates and serial learning do not differ in the basic assumptions or model processes, but only in the degree to which certain list transitions are isolated from the rest of the list. Thus, the Isolation Principle is a unifying principle.

The generality of the Isolation Principle is evident when one considers that it places quite different constraints on positional than on chaining models. In the chaining model, the high correlation in PAL derived from the fact that the stored forward and backward association terms were susceptible to identical noise terms. In the positional model, there were no stored forward and backward terms per se. Rather, each item was stored in a separate operation, and had its own independent noise term. However, at retrieval, forward and backward probes accumulated the same noise/variability terms from the probe and target items. In PAL, those noise terms dominated, but in SL, noise accumulated from competing list items differed between probe directions.

3.1. The empirical dissociation between PAL and SL

Whereas high forward–backward probe correlation is found for probes of pairs, supporting associative symmetry (Kahana, 2002, Rizzuto & Kahana, 2000, 2001), here I report that this correlation is substantially lower for probes of previously learned serial lists. While this comparison is across experiments with many procedural differences, the dissociation cannot be explained by two prominent differences—namely, the large value of $\Delta\mathcal{Q}$ obtains even for participants with high test/re-test reliability (Fig. 1a), and across levels of learning between participants (Fig. 1b), between lists (Fig. 1c) and between serial positions within lists (Fig. 1d). These analyses also showed that the large value of $\Delta\mathcal{Q}$ in SL generalizes across level of learning. This parallels the finding that the small-valued $\Delta\mathcal{Q}$ in PAL does not interact with number of presentations (Rizzuto and Kahana, 2001).

3.2. General conditions for the Isolation Principle

The following conditions were necessary for the Isolation Principle to produce near-perfect correlations in PAL and imperfect correlations in SL:

(1) In the PAL regime, forward and backward probes must be susceptible to the same sources of variability when probing a given pair (or list transition). This will most likely include encoding variability of the pair of interest, but could potentially also include some extra-pair noise.
(2) If the noise accumulates in a different order depending on probe direction (as in the case of the positional model), the noise terms must commute, at least in the PAL regime.
(3) Noise must vary across the list to introduce the possibility of reducing the correlation between forward and backward probes.
(4) Extra-pair noise terms must be somewhat different depending on probe direction. This is necessary to obtain the moderate correlation in SL.
(5) There must be a parameter that, at some values, "isolates" a given pair from the rest of the list in the sense that the noise terms due to extra-pair competition become very small.

3.3. Forgetting

The near-perfect correlations obtained in the PAL data suggests that there is little noise between successive tests, or that the noise was perfectly correlated between tests. In developing the models, we assumed the former, and implement no noise between Test 1 and Test 2, because the main goal of this work is to demonstrate that the Isolation Principle can produce dissociations like the one that has been observed. However, it will ultimately be important to fit richer data in which substantial noise is introduced between successive tests; for example, by separating the tests by a large amount of time or by considering the low-$\mathcal{Q}_{\text{Same}}$ participants in Fig. 1a. One can make the conditions more generally applicable. Namely, the forgetting terms must influence the noise terms in a way that does not differentially affect probes according to direction. For the positional coding model presented here, adding a noise term to the item–position and position–item association terms would work, as long as the same noise was added to both terms for a given list item. For the chaining model, one could similarly add noise to the forward and backward association terms, as long as the same noise term was added to both terms for a given list pair. Clearly, mechanisms for forgetting that substantially altered the degree of isolation, whether positional or associative, would alter the correlation between forward and backward probed recall.

3.4. Symmetric versus asymmetric mean accuracy

Kahana (2000) showed that correlated models can yield asymmetric accuracy, and independent models can yield symmetric accuracy. For this reason, in a mathematical sense, accounting for symmetry/asymmetry in the expectations is a distinct problem from accounting for the correlations. Still, however one chooses to account for symmetric and asymmetric means, one needs to ensure that this will not undermine how well the model fits the correlations. In this spirit, one can place constraints on accounts of mean accuracy.

According to the Isolation Principle, within-pair noise terms dominate probed recall in PAL, while in SL, extra-pair noise terms are non-negligible. For PAL to have symmetric mean accuracy, the expectations of the within-pair terms must be identical for forward and backward probes. Therefore, the asymmetry in SL must be a consequence of unequal expectations of the extra-pair terms.

As a concrete example of a mechanism that could produce asymmetry in SL, consider the hypothesis that at times of low confidence, participants guess using a free-recall-type strategy. This type of guessing might be driven by the similarity of an item's positional code to the end-of-list position, favoring later list items over early list items. The probability of retrieving the target by chance on the forward probe would be slightly higher than on the backward probe in SL, when the similarity of positional codes between an item and the end-of-list position is substantially different between any two adjacent items. In PAL, similarity to end-of-list position would not change substantially across the pair; hence, probability of a lucky guess would be comparable (and correlated) between forward and backward probes. Another consequence of this type of guessing would be a forward advantage for intruded list items that is comparable for forward and backward probes, a pattern suggested by probed SL data (Kahana and Caplan, 2002).

An example for the positional coding model is to assume that positional similarity does not commute. In other words, $S(t_l, t_k) > S(t_k, t_l)$ for $k > l$. Asymmetry in similarity judgements are common (cf. Medin et al., 1993). In the case of SL, items are evenly spaced across serial position; hence, position will be a better cue for items with subsequent positional codes than for items with earlier positional codes. However, in the case of PAL, positional similarity for the two items within a pair is already nearly perfect, so the non-commutative nature of similarity will have a vanishingly small effect.

3.5. Rehearsal

An alternate hypothesis is that rehearsal processes in both SL and PAL produce correlated forward and backward probed recall. If a participant rehearses a pair repeatedly, as in "A-B-A-B-A-B-A-B", then they are rehearsing the forward pair $\mathbf{A}-\mathbf{B}$ as often as the backward pair, $\mathbf{B}-\mathbf{A}$. For PAL, this should result in correlated forward and backward probed recall within pairs because the forward and backward associations (or positional codes) are rehearsed at similar times, and are therefore susceptible to the same noise. In contrast, during a SL task, rehearsal might be less symmetric, but also, a given item could be rehearsed with more than one other item (see the discussion of double-function lists below), and so neighboring items will be susceptible to different sources of noise. One could perform a strong empirical test of the Rehearsal Hypothesis against the Isolation Hypothesis (see the following discussion on the continuum of isolation for an example). However, the failure to find a change in associative symmetry as a function of degree of learning (including, presumably, amount of rehearsal) casts doubt on this alternate account (Kahana, 2000; Rizzuto and Kahana, 2001, and Table 2 of this manuscript).

3.6. A continuum of isolation

I have suggested that PAL and SL lie at ends of a continuum of isolation, where paired associates are very isolated from the rest of the list and in serial lists, pairs of neighboring items are not at all isolated from the rest of the list. This suggests that with careful experimental design, one should be able to produce *intermediate* degrees of isolation, thereby producing correlations between forward and backward recall that lie between the perfect correlations found in PAL and the moderate correlations found in SL. This may in fact occur in typical serial learning experiments, simply due to participants' encoding associations with variable strengths in the case of an associative chaining model, or spacing list items' positional codes with some variability.

One way to manipulate this would be to present lists with serial recall instructions, with a grouped presentation schedule (along the lines of chunking experiments; e.g., Bower, 1969; Brannon, 1997)—namely, $ISI_{\text{within-pair}} \gg ISI_{\text{between-pair}}$. The prediction is that probing within pairs should result in greater correlation than probing between pairs. However, "pairs" are really adjacent items in a serial list, so competition from extra-pair items should reduce the within-pair correlation relative to standard PAL presentation.

One should further be able to dissociate within- from between-pair correlations by presenting an intralist distractor task between pairs of items in the list (i.e., *filled* as opposed to *unfilled* grouping). The Isolation Hypothesis would predict that within-pair correlations should be greater in the filled than in the unfilled condition, while between-pair correlations should show the opposite pattern. A similar procedure was in fact used by Glenberg and Swanson (1986) to test a temporal distinctiveness theory of free recall. Their paradigm differs significantly from our proposed paradigm, in that instructions and tests were for free rather than serial recall. It is noteworthy that they accounted for long-term modality and long-term recency effects by use of an adaptable-size temporal search set. The way in which this mechanism produces interference has the same flavor as the way in which manipulations of isolation influence the degree of interference from extra-pair list items.

Interestingly, the Rehearsal Hypothesis makes the exact opposite prediction. According to the Rehearsal Hypothesis, it is rehearsal that *produces* correlated forward and backward probed recall. Therefore, the filled condition, because it suppresses this process, should produce *lower* within-pair correlations than the unfilled condition.

Finally, the Isolation Principle implies that very short lists of length 3 (triples) should already begin to dissociate from pairs in terms of forward–backward probe correlation, and this should be entirely due to the addition of an item in the case of triples (Caplan, 2004).

3.7. *Double-function paired associates lists*

In double-function PAL designs, each item acts as the "A" item in one paired associate and as the "B" item in another (Primoff, 1938; Slamecka, 1976; Stark, 1968). While this task has similarities to SL, transfer from double-function PAL to SL is only moderate (Slamecka, 1976). Furthermore, forward and backward probes show symmetric probability of recall on average (Horowitz et al., 1966). However, according to the Isolation Hypothesis, forward and backward recall should nonetheless be only moderately correlated, because there will be substantial competition from items from other pairs, uncorrelated between forward and backward probes. For example, consider the pairs **A**–**B**, **B**−**C** and **C**−**D**. Now, probe the middle pair. First, in the forward direction, probing with **B** and asking for **C**. In this case, the dominant competitor is **A**, the other associate of the probe. In the backward probe, probing with **C** and asking for **B**, the dominant competitor is **D**, the other associate of the probe. Hence, the near-perfect correlation found in single-function PAL should be significantly decorrelated in the case of double-function lists, even in the presence of symmetric probability of recall. According to our approach, double-function lists show only moderate correlations for the same reason as serial lists.

3.8. *PAL with asymmetric accuracy*

In early PAL research, experimenters presented paired associates wherein the "A" and "B" items were of different material types. Lockhart (1969) demonstrated that one could produce asymmetry by varying the abstractness of the "A" compared to that of the "B" items. He provided evidence that abstractness determines the effectiveness of words as cues. When abstractness was controlled for, he could recover symmetric mean accuracy. However, according to the Isolation Hypothesis, even when average accuracy is asymmetric, probed recall of such paired-associates lists should be nearly perfectly correlated.

An example of this can be seen in the treatment of the associative chaining model. As Fig. 2c illustrates, even when backward associations are weaker than forward associations, the correlation dependence on isolation is unaffected. Thus, from a theoretical standpoint, it is at least as reasonable to expect that pairs with asymmetric mean accuracy will have high forward/backward correlations as that they should have lower correlations.

4. Summary

I first presented empirical dissociations between PAL and SL. I then introduced the Isolation Principle as a means of accounting for these differences, treating PAL and SL within the same theoretical framework. This approach suggests a continuum of paradigms, modifiable by a single continuous parameter. I showed how the Isolation Principle may be integrated into both positional and associative chaining models. In the course of these derivations I uncovered constraints that must be satisfied to account for the observed data patterns. Finally, although the Isolation Principle can be implemented in very different classes of models, it is specific enough to be tested empirically.

Acknowledgments

I thank Mike Kahana, Kelly Addis and Dan Rizzuto for helping to set up the problem and for feedback on the manuscript, and Mackenzie Glaholt for additional feedback. Supported by National Institutes of Health Grant MH55687.

Appendix. Correlation for the positional coding model

First, we will use the following solutions to infinite series (solved with the help of the MATLAB Symbolic Toolbox, The Mathworks, Inc.):

Linear summations:

$$\begin{aligned}\Sigma_{\text{lin}} &= \sum_{y\neq x,x+1} S(t_{x+1},t_y)\\ &= \sum_{v=0}^{\infty} e^{-(\Delta+v)/\tau}[\text{for } y=x+2,x+4,\ldots]\\ &\quad+\sum_{v=0}^{\infty} e^{-(1+v)/\tau}[\text{for } y=x+3,x+5,\ldots]\\ &\quad+\sum_{v=0}^{\infty} e^{-(1+v)/\tau}[\text{for } y=x-1,x-3,\ldots]\end{aligned}$$

$$+\sum_{v=0}^{\infty} e^{-(2-\Delta+v)/\tau}[\text{for } y=x-2,x-4,\ldots]$$
$$=\frac{(e^{-(\Delta-1)/\tau}+1)^2 e^{(\Delta-1)/\tau}}{e^{1/\tau}-1}. \quad (15)$$

Quadratic summations:

$$\Sigma_{\text{quad}} = \sum_{y\neq x,x+1} S(t_{x+1},t_y)^2$$
$$=\frac{(e^{-2(\Delta-1)/\tau}+1)^2 e^{2(\Delta-1)/\tau}}{e^{2/\tau}-1}. \quad (16)$$

Cross-term summations:

$$\Sigma_{\text{cross}} = \sum_{y\neq x,x+1}\ \sum_{z\neq x,x+1,y} S(t_{x+1},t_y)S(t_{x+1},t_z)$$
$$= \mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{D}+\mathrm{E}+\mathrm{F}$$

where

$$\begin{cases} \mathrm{A}: & y>x+1, z>y,\\ \mathrm{B}: & z>x+1, y>z,\\ \mathrm{C}: & y<x-1, z<y,\\ \mathrm{D}: & z<x-1, y<z,\\ \mathrm{E}: & y>x+1, z<x-1,\\ \mathrm{F}: & y<x-1, z>x+1. \end{cases}$$

Due to symmetry, $\mathrm{A}=\mathrm{B}, \mathrm{C}=\mathrm{D}, \mathrm{E}=\mathrm{F}$

$$\mathrm{A} = \sum_{y>x+1}\sum_{z>y} S(t_{x+1},t_y)S(t_{x+1},t_z)$$
$$=\sum_{v=0}^{\infty}\sum_{w=0}^{\infty} e^{-(\Delta+v)/\tau}$$
$$\times(e^{-(1+v+w)/\tau}+e^{-(1+\Delta+v+w)/\tau})$$
$$+e^{-(1+v)/\tau}(e^{-(1+\Delta+v+w)/\tau}+e^{-(2+v+w)/\tau}),$$

$$\mathrm{C} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty} e^{-(1+v)/\tau}(e^{-(2-\Delta+v+w)/\tau}+e^{-(2+v+w)/\tau})$$
$$+e^{-(2-\Delta+v)/\tau}(e^{-(2+v+w)/\tau}+e^{-(3-\Delta+v+w)/\tau}),$$

$$\mathrm{E} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty}(e^{-(\Delta+v)/\tau}+e^{-(1+w)/\tau})$$
$$\times(e^{-(1+v)/\tau}+e^{-(2-\Delta+w)/\tau}).$$

Substituting these expressions and solving the infinite series,

$$\Sigma_{\text{cross}} = 2\mathrm{A}+2\mathrm{C}+2\mathrm{E}$$
$$=2[4e^{-3/\tau}+2e^{(\Delta-4)/\tau}+2e^{-(2+\Delta)/\tau}$$
$$+2e^{-2/\tau}+2e^{(\Delta-3)/\tau}$$
$$+2e^{-(1+\Delta)/\tau}+e^{(2\Delta-5)/\tau}$$
$$+e^{-(2\Delta+1)/\tau}]/[(e^{-1/\tau}-1)(e^{-2/\tau}-1)]. \quad (17)$$

Adjacent term summations:

$$\Sigma_{\text{adj}} = \sum_{y\neq x,x+1} S(t_{x+1},t_y)S(t_x,t_y)$$
$$=\sum_{v=0}^{\infty} e^{-(\Delta+v)/\tau}e^{-(1+v)/\tau}[\text{for } y=x+2,x+4,\ldots]$$
$$+\sum_{v=0}^{\infty} e^{-(1+v)/\tau}e^{-(2-\Delta+v)/\tau}[\text{for } y=x+3,x+5,\ldots]$$
$$+\sum_{v=0}^{\infty} e^{-(1+v)/\tau}e^{-(\Delta+v)/\tau}[\text{for } y=x-1,x-3,\ldots]$$
$$+\sum_{v=0}^{\infty} e^{-(2-\Delta+v)/\tau}e^{-(1+v)/\tau}[\text{for } y=x-2,x-4,\ldots]$$
$$=2\,\frac{(e^{2/\tau}+e^{2\Delta/\tau})e^{-(1+\Delta)/\tau}}{e^{2/\tau}-1}. \quad (18)$$

Adjacent cross-term summations:

$$\Sigma_{\text{adj-cross}} = \sum_{y\neq x,x+1}\ \sum_{z\neq x,x+1,y} S(t_{x+1},t_y)S(t_x,t_z)$$
$$= \mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{D}+\mathrm{E}+\mathrm{F}$$

where

$$\begin{cases} \mathrm{A}: & y>x+1, z>y,\\ \mathrm{B}: & z>x+1, y>z,\\ \mathrm{C}: & y<x-1, z<y,\\ \mathrm{D}: & z<x-1, y<z,\\ \mathrm{E}: & y>x+1, z<x-1,\\ \mathrm{F}: & y<x-1, z>x+1. \end{cases}$$

$$\mathrm{A} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty} e^{-(\Delta+v)/\tau}$$
$$\times(e^{-(2-\Delta+v+w)/\tau}+e^{-(2+v+w)/\tau})$$
$$+e^{-(1+v)/\tau}(e^{-(2+v+w)/\tau}+e^{-(3-\Delta+v+w)/\tau}),$$

$$\mathrm{B} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty} e^{-(1+w)/\tau}$$
$$\times(e^{-(1+v+w)/\tau}+e^{-(1+\Delta+v+w)/\tau})$$
$$+e^{-(2-\Delta+w)/\tau}(e^{-(1+\Delta+v+w)/\tau}+e^{-(2+v+w)/\tau}),$$

$$\mathrm{C} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty} e^{-(1+v)/\tau}$$
$$\times(e^{-(1+v+w)/\tau}+e^{-(1+\Delta+v+w)/\tau})$$
$$+e^{-(2-\Delta+v)/\tau}(e^{-(1+\Delta+v+w)/\tau}+e^{-(2+v+w)/\tau}),$$

$$\mathrm{D} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty} e^{-(\Delta+w)/\tau} \times (e^{-(2-\Delta+v+w)/\tau} + e^{-(2+v+w)/\tau}) + e^{-(1+w)/\tau}(e^{-(2+v+w)/\tau} + e^{-(3-\Delta+v+w)/\tau}),$$

$$\mathrm{E} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty} (e^{-(\Delta+v)/\tau} + e^{-(1+v)/\tau}) \times (e^{-(\Delta+w)/\tau} + e^{-(1+w)/\tau}),$$

$$\mathrm{F} = \sum_{v=0}^{\infty}\sum_{w=0}^{\infty} (e^{-(1+v)/\tau} + e^{-(2-\Delta+v)/\tau}) \times (e^{-(1+w)/\tau} + e^{-(2-\Delta+w)/\tau}).$$

Substituting these expressions and solving the infinite series,

$$\begin{aligned}\Sigma_{\text{adj-cross}} = [&e^{(3-2\Delta)/\tau} + 6e^{(1-\Delta)/\tau} + e^{2(1-\Delta)/\tau} \\ &+ 2e^{(2-\Delta)/\tau} + 6e^{1/\tau} + 6e^{(\Delta-1)/\tau} \\ &+ 6 + 2e^{\Delta/\tau} + e^{(2\Delta-1)/\tau} \\ &+ e^{2(\Delta-1)/\tau}]/[(e^{1/\tau}+1)^2(e^{1/\tau}+1)].\end{aligned}$$

Derivations for variance and covariance: To derive ρ_{FB}, we need expressions for the variance and covariance between ξ_{F} and ξ_{B}. If we define the following terms:

$$\mathrm{T}_1 = \gamma_x\gamma_{x+1},$$
$$\mathrm{T}_{2f} = \sum_{y\neq x,x+1} S(t_{x+1}, t_y)\gamma_{x+1}\gamma_y,$$
$$\mathrm{T}_{2b} = \sum_{y\neq x,x+1} S(t_x, t_y)\gamma_x\gamma_y,$$

$$\mu = \mathrm{E}[\xi_{\mathrm{B}}] = \mathrm{E}[\xi_{\mathrm{F}}] = 1 - \sum_{y\neq x,x+1} S(t_{x+1}, t_y) = 1 - \Sigma_{\text{lin}}$$

then

$$\mathrm{var}[\xi_{\mathrm{B}}] = \mathrm{var}[\xi_{\mathrm{F}}] = \mathrm{E}[(\mathrm{T}_1 - \mathrm{T}_{2f} - \mu)^2],$$
$$\mathrm{cov}[\xi_{\mathrm{F}}, \xi_{\mathrm{B}}] = \mathrm{E}[(\mathrm{T}_1 - \mathrm{T}_{2f} - \mu)(\mathrm{T}_1 - \mathrm{T}_{2b} - \mu)].$$

We need to solve for the expectations of the corresponding products of these terms, making use of the following:

$$\mathrm{E}[\gamma_x^2\gamma_y^2] = \sigma^4 + 2\sigma^2 + 1, \quad x\neq y,$$
$$\mathrm{E}[\gamma_x^2\gamma_y\gamma_z] = \sigma^2 + 1, \quad x\neq y\neq z.$$

Common components:

$$\mathrm{E}[\mathrm{T}_1^2] = \sigma^4 + 2\sigma^2 + 1,$$
$$\mathrm{E}[\mu^2] = \mu^2 = (1 - \Sigma_{\text{lin}})^2,$$
$$\mathrm{E}[\mathrm{T}_1\mathrm{T}_{2b}] = \mathrm{E}[\mathrm{T}_1\mathrm{T}_{2f}] = (\sigma^2+1)\Sigma_{\text{lin}},$$
$$\mathrm{E}[\mathrm{T}_1\mu] = \mu = 1 - \Sigma_{\text{lin}},$$
$$\mathrm{E}[\mathrm{T}_{2f}\mu] = \mathrm{E}[\mathrm{T}_{2b}\mu] = \mu\Sigma_{\text{lin}} = \Sigma_{\text{lin}} - \Sigma_{\text{lin}}^2.$$

Variance-only components:

$$\mathrm{E}[\mathrm{T}_{2f}^2] = (\sigma^4 + 2\sigma^2 + 1)\Sigma_{\text{quad}} + (\sigma^2+1)\Sigma_{\text{cross}}.$$

Covariance-only components:

$$\mathrm{E}[\mathrm{T}_{2f}\mathrm{T}_{2b}] = (\sigma^2+1)\Sigma_{\text{adj}} + \Sigma_{\text{adj-cross}}.$$

Variance:

$$\begin{aligned}\mathrm{var}[\xi_{\mathrm{B}}] &= \mathrm{var}[\xi_{\mathrm{F}}] \\ &= \sigma^4 + 2\sigma^2 + 1 + (1-\Sigma_{\text{lin}})^2 \\ &\quad - 2(1-\Sigma_{\text{lin}}) - 2(\sigma^2+1)\Sigma_{\text{lin}} \\ &\quad + 2(\Sigma_{\text{lin}} - \Sigma_{\text{lin}}^2) \\ &\quad + (\sigma^4+2\sigma^2+1)\Sigma_{\text{quad}} + (\sigma^2+1)\Sigma_{\text{cross}} \\ &= \sigma^4 + 2\sigma^2 - 2\sigma^2\Sigma_{\text{lin}} - \Sigma_{\text{lin}}^2 \\ &\quad + (\sigma^4+2\sigma^2+1)\Sigma_{\text{quad}} + (\sigma^2+1)\Sigma_{\text{cross}}.\end{aligned} \tag{19}$$

Covariance:

$$\begin{aligned}\mathrm{cov}[\xi_{\mathrm{F}}, \xi_{\mathrm{B}}] &= \sigma^4 + 2\sigma^2 + 1 + (1-\Sigma_{\text{lin}})^2 - 2(1-\Sigma_{\text{lin}}) \\ &\quad - 2(\sigma^2+1)\Sigma_{\text{lin}} + 2(\Sigma_{\text{lin}} - \Sigma_{\text{lin}}^2) \\ &\quad + (\sigma^2+1)\Sigma_{\text{adj}} + \Sigma_{\text{adj-cross}}\end{aligned} \tag{20}$$

$$\begin{aligned}&= \sigma^4 + 2\sigma^2 - 2\sigma^2\Sigma_{\text{lin}} - \Sigma_{\text{lin}}^2 \\ &\quad + (\sigma^2+1)\Sigma_{\text{adj}} + \Sigma_{\text{adj-cross}}.\end{aligned} \tag{21}$$

References

Addis, K. M., & Kahana, M. J. (in preparation). Positional uncertainty in serial learning.

Asch, S. E., & Ebenholtz, S. M. (1962). The principle of associative symmetry. *Proceedings of the American Philosophical Society*, *106*(2), 135–163.

Baddeley, A. D. (1968). How does acoustic similarity influence short-term memory? *Quarterly Journal of Experimental Psychology*, *20*(3), 249–264.

Borsellino, A., & Poggio, T. (1971). Holographic aspects of temporal memory and optomotor responses. *Kybernetik*, *10*(1), 58–60.

Bower, G. H. (1969). Chunks as interference units in free recall. *Journal of Verbal Learning and Verbal Behavior*, *8*, 610–613.

Brannon, E. M. (1997). *Chunking in memory*: *Toward an operational definition of a chunk*. Master's thesis, Columbia University, New York.

Brown, G. D. A., Preece, T., & Hulme, C. (2000). Oscillatory-based memory for serial order. *Psychological Review*, *107*(1), 127–181.

Burgess, N., & Hitch, G. J. (1999). Memory for serial order: A network model of the phonological loop and its timing. *Psychological Review*, *106*, 551–581.

Caplan, J. B. (2004). Unifying models of paired associates and serial learning: Insights from simulating a chaining model. *NeuroComputing*, *58–60*, 739–743.

Ebbinghaus, H. (1885/1913). *Memory: A contribution to experimental psychology*. New York: Teachers College, Columbia University.

Giurintano, S. L. (1973). Serial learning process: Test of chaining, position, and dual-process hypotheses. *Journal of Experimental Psychology*, *97*(2), 154–157.

Glenberg, A. M., & Swanson, N. G. (1986). A temporal distinctiveness theory of recency and modality effects. *Journal of Experimental Psychology: Learning Memory, and Cognition*, *12*(1), 3–15.

Henson, R. N. A. (1998). Short-term memory for serial order: The start-end model. *Cognitive Psychology*, *36*, 73–137.

Henson, R. N. A., Norris, D. G., Page, M. P. A., & Baddeley, A. D. (1996). Unchained memory: Error patterns rule out chaining models of immediate serial recall. *Quarterly Journal of Experimental Psychology*, *49A*(1), 80–115.

Hintzman, D. L. (1993). On variability, Simpson's paradox, and the relation between recognition and recall: Reply to Tulving and Flexser. *Psychological Review*, *100*(1), 143–148.

Hockley, W. E. (1992). Item versus associative information: Further comparisons of forgetting rates. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *18*, 1321–1330.

Horowitz, L. M., Brown, Z. M., & Weissbluth, S. (1964). Availability and the direction of associations. *Journal of Experimental Psychology*, *68*(6), 541–549.

Horowitz, L. M., Norman, S. A., & Day, R. S. (1966). Availability and associative symmetry. *Psychological Review*, *73*(1), 1–15.

Howard, M. W., & Kahana, M. J. (2002). A distributed representation of temporal context. *Journal of Mathematical Psychology*, *46*(3), 269–299.

Humphreys, M. S., Bain, J. D., & Pike, R. (1989). Different ways to cue a coherent memory system: A theory for episodic, semantic, and procedural tasks. *Psychological Review*, *96*, 208–233.

Humphreys, M. S., Pike, R., Bain, J. D., & Tehan, G. (1989). Global matching: A comparison of the SAM, Minerva II, Matrix, and TODAM models. *Journal of Mathematical Psychology*, *33*, 36–67.

Kahana, M.J. (2000). Contigency analyses of memory. In E. Tulving, & F. I. M. Craik (Eds.), *The Oxford handbook of memory* (Chap. 4, pp. 59–72). Oxford: Oxford University Press.

Kahana, M. J. (2002). Associative symmetry and memory theory. *Memory & Cognition*, *30*(6), 823–840.

Kahana, M. J., & Caplan, J. B. (2002). Associative asymmetry in probed recall of serial lists. *Memory & Cognition*, *30*(6), 841–849.

Köhler, W. (1947). *Gestalt psychology*. New York: Liveright.

Lee, C. L., & Estes, W. K. (1977). Order and position in primary memory for letter strings. *Journal of Verbal Learning and Verbal Behavior*, *16*, 395–418.

Lewandowsky, S., & Farrell, S. (2000). A redintegration account of the effects of speech rate, lexicality, and word frequency in immediate serial recall. *Psychological Research*, *63*, 163–173.

Lewandowsky, S., & Murdock, B. B. (1989). Memory for serial order. *Psychological Review*, *96*, 25–57.

Lockhart, R. S. (1969). Retrieval asymmetry in the recall of adjectives and nouns. *Journal of Experimental Psychology*, *79*(1), 12–17.

Loftus, G. R., & Masson, M. E. J. (1994). Using confidence intervals in within-subject designs. *Psychonomic Bulletin & Review*, *1*(4), 476–490.

Luce, R. D. (1959). Detection and recognition. In R. D. Luce, R. R. Bush, & E. Galanter (Eds.), *Handbook of mathematical psychology* (pp. 103–189). New York: Wiley.

Maisto, A. A., & Ward, L. C. (1976). A test of the associative chaining hypothesis by serial-recall and serial-anticipation procedures. *Journal of General Psychology*, *94*, 75–83.

McNaughton, B. L., & Morris, R. G. (1987). Hippocampal synaptic enhancement and information storage within a distributed memory system. *Trends in Neurosciences*, *10*, 408–415.

Medin, D. L., Goldstone, R. L., & Gentner, D. (1993). Respects for similarity. *Psychological Review*, *100*(2), 254–278.

Mensink, G.-J., & Raaijmakers, J. G. W. (1988). A model for interference and forgetting. *Psychological Review*, *95*(4), 434–455.

Murdock, B. B. (1962). Direction of recall in short-term memory. *Journal of Verbal Learning and Verbal Behavior*, *1*, 119–124.

Murdock, B. B. (1965). Associative symmetry and dichotic presentation. *Journal of Verbal Learning and Verbal Behavior*, *4*, 222–226.

Murdock, B. B. (1966). Forward and backward associations in paired associates. *Journal of Experimental Psychology*, *71*(5), 732–737.

Murdock, B. B. (1967). Distractor and probe techniques in short-term memory. *Canadian Journal of Experimental Psychology*, *21*(1), 25–36.

Murdock, B. B. (1974). *Human memory: Theory and data*. New York: Wiley.

Murdock, B. B. (1979). Convolution and correlation in perception and memory. In L. G. Nilsson (Ed.), *Perspectives in memory research: Essays in honor of Uppsala University's 500th Anniversary* (pp. 105–119). Hillsdale, NJ: Lawrence Erlbaum Associates.

Murdock, B. B. (1993). TODAM2: A model for the storage and retrieval of item, associative, and serial-order information. *Psychological Review*, *100*(2), 183–203.

Murdock, B. B., & Franklin, P. E. (1984). Associative and serial-order information: Different modes of operation? *Memory & Cognition*, *12*(3), 243–249.

Murdock, B. B., & Hockley, W. E. (1989). Short-term memory for associations. In G. H. Bower (Ed.), *The psychology of learning and motivation: Advances in research and theory*, Vol. 24 (pp. 71–108). San Diego: Academic Press Inc.

Page, M. P. A., & Norris, D. (1998). The primacy model: A new model of immediate serial recall. *Psychological Review*, *105*, 761–781.

Primoff, E. (1938). Backward and forward associations as an organizing act in serial and in paired-associate learning. *Journal of Psychology*, *5*, 375–395.

Rizzuto, D. S., & Kahana, M. J. (2000). Associative symmetry vs. independent associations. *NeuroComputing*, *32–33*, 973–978.

Rizzuto, D. S., & Kahana, M. J. (2001). An autoassociative neural network model of paired-associate learning. *Neural Computation*, *13*, 2075–2092.

Slamecka, N. J. (1976). An analysis of double-function lists. *Memory & Cognition*, *4*(5), 581–585.

Stark, K. (1968). Transfer from serial to paired-associate learning: A reappraisal. *Journal of Verbal Learning and Verbal Behavior*, *7*, 20–30.

Wickelgren, W. A. (1966). Associative intrusions in short-term recall. *Journal of Experimental Psychology*, *72*(6), 853–858.

Wolford, G. (1971). Function of distinct associations for paired-associate performance. *Psychological Review*, *78*(4), 303–313.

Woodward, A. E., & Murdock, B. B. (1968). Positional and sequential probes in serial learning. *Canadian Journal of Psychology*, *22*(2), 131–138.

Woodworth, R. S. (1915). A revision of imageless thought. *Psychological Review*, *22*(1), 1–27.

Young, R. K. (1968). Serial learning. In T. R. Dixon, & D. L. Horton (Eds.), *Verbal behavior and general behavior theory* (pp. 122–148). Englewood Cliffs, NJ: Prentice-Hall.