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Discussion:

Of Selection Operators and Semanticists

As anyone who has ever seriously attempted to analyze the semantico-epistemological status of scientific theories has soon discovered, it is not easy to reconcile the belief that theoretical terms (i.e., terms which cannot be explicitly defined in the observation language) have genuine cognitive properties with the empiricist tenet that all knowledge derives from experience. Even if it be granted that knowledge can originate in experience without being *about* experience, it still remains to develop a coherent metalinguistic account of the truth-conditions of theoretical propositions and the designata of denotative expressions in the theory language. In a recent article, "On the use of Hilbert's ϵ-operator in scientific theories", Carnap (1961) has broadened his analysis of theories to include provision for the referential properties of individual theoretical terms. But while his account focuses more closely than ever upon the fundamental semantical problems of a theory language, his specific proposals amount to a repudiation of the very logical empiricism which he has so strongly championed these many years. In what follows, I use Carnap's notation and terminology throughout (including a pejorative sense of the term "metaphysics" borrowed from Carnap's earlier writings).

Let '$\Phi(t, o)$' be a theory which ascribes a predicate 'Φ', composed only of logical terms, to the n-tuple 't' ($t = \langle t_1, \ldots, t_n \rangle$) of theoretical terms and the m-tuple 'o' ($o = \langle o_1, \ldots, o_m \rangle$)" of observational terms. (For grammatical simplicity, t will be spoken of as though $n = 1$.) As in previous writings, Carnap assumes that the factual content of a theory is given by its Ramsey-sentence—i.e., that

$$\Phi(t, o) \equiv (\exists u)\, \Phi(u, o)$$

is A-true.[1] Then, says Carnap, we can analyze the theory $\Phi(t, o)$' into a wholly factual component

$$(\exists u)\, \Phi(u, o)$$

[1] Carnap has yet to provide arguments which support this assumption. Using a plausible formalization of some of Carnap's major semantical tenets, I have elsewhere (Rozeboom, 1960) developed a proof of it which is rigorous through the penultimate step. However, the final move to the conclusion that a theory is equivalent to its Ramsey-sentence apparently requires a crucial assumption (*ibid.* fn. 7) which is incompatible with Carnap's present stand. Simply to stipulate that '$\Phi(t, o) \equiv (\exists u)\, \Phi(u, o)$' (or equivalently '$\Phi(t, o) \supset (\exists u)\, \Phi(u, o)$') is a meaning postulate, as proposed by a reviewer of this note, would merely beg the question. For what is at issue is the problematic semantical status conferred upon theoretical term 't' by acceptance of theory '$\Phi(t, o)$', and even if it be agreed that the meaning so acquired by 't' can be expressed by some meaning postulate '*M(t)*' it still remains to show that '*M(t)*' is equivalent to $\Phi(t, o) \equiv (\exists u)\, \Phi(u, o)$.

and a wholly analytic meaning postulate

$$t = \epsilon_u \Phi(u, o),$$

in which 'ϵ' is a selection-operator such that for any sentential function 'Fx', '$\epsilon_x Fx$' designates some entity which satisfies 'Fx' so long as such an entity exists.

Now, the beauty of the ϵ-operator is that since

$$(\exists x)Fx \equiv F(\epsilon_x Fx)$$

obtains by axiomatic stipulation, it generates statements which are apparently equivalent in force to existence-statements, but in which the existentially bound variable is replaced by a designating constant. Hence if $t = \epsilon_u \Phi(u, o)$, the theory '$\Phi(t, o)$' remains factually equivalent to its Ramsey-sentence even while 't' *designates* some entity which may lie outside of the observation language's referential scope. This device seems entirely acceptable to me *so far as Carnap's formal axiomatization takes it*, for to this extent it yields results which are in close accord with my own conclusions (Rozeboom, 1962, 1960). The basic problems still remain, however; only now they concern the semantical interpretation of the selection-operator. In particular, questions which need to be answered are (*I*) what, specifically, is designated by the (well-formed) expression '$\epsilon_x Fx$', and (*II*) what is the truth-value of a formally undecidable sentence of form '$G(\epsilon_x Fx)$'?[2] Carnap's present answers are bound to prove indigestable to an empirically queasy philosophical stomach.

With respect to question (*I*), suppose that 'Fx' is a sentential function which is satisfied by exactly two entities, a and b ($a \neq b$). There are then five logically exhaustive alternatives concerning the possible designata of '$\epsilon_x Fx$': (1) '$\epsilon_x Fx$' designates nothing. (2) '$\epsilon_x Fx$' designates some entity which does not satisfy 'Fx'. (3) '$\epsilon_x Fx$' designates a and a only. (4) '$\epsilon_x Fx$' designates b and b only. (5) '$\epsilon_x Fx$' designates both a and b, and these only. Since we have stipulated that 'Fx' has satisfiers, possibilities (1) and (2) are ruled out by the intended definition of the ϵ-operator. But does '$\epsilon_x Fx$' then designate a to the exclusion of b, or b to the exclusion of a? If so, two further possibilities present themselves: (*i*) The selection-operator provides a criterion by which can be determined which of the various satisfiers of 'Fx' is the one designated by '$\epsilon_x Fx$'. This alternative does not seem to get anywhere, for any additional identifying criteria, theoretical or observational, built into '$\epsilon_x Fx$' could be added to 'Fx' to form an enriched predicate 'F^*x' such that $\epsilon_x Fx = \epsilon_x F^*x$ and where '$\epsilon_x F^*x$' does *not* afford any criterion for which of various satisfiers of 'F^*x' it designates. In any case, Carnap explicitly rejects this possibility. However, the other alternative under (3) or (4)

[2] (*I*) is a special case of (*II*) if the language under consideration contains designators for the various possible designata of '$\epsilon_x Fx$'. Thus if 'e' designates an entity e, whether or not '$\epsilon_x Fx$' designates e is equivalent to whether or not "$\epsilon_x Fx = e$" is true.

is that (*ii*) the selection-operator does, in fact, single out one satisfier of 'Fx' as the unique designatum of '$\epsilon_x Fx$' even though there is no way to determine, not even in principle, which one $\epsilon_x Fx$ is. This appears to be the interpretation now favored by Carnap, but it entails the very sort of carefree metaphysics that he has consistently abjured. If an expression E designates entity e_1 but not e_2, then surely there must be *something* about the linguistic role of E and the respective natures of e_1 and e_2 which is responsible for this selective designation. But if such a condition exists, there is no reason why we couldn't be aware of it, and hence no reason why in principle the referent of '$\epsilon_x Fx$' could not be determined. Putting this another way, (*ii*) says that although '$\epsilon_x Fx$' singles out exactly one of the several satisfiers of 'Fx', it does not do so according to any knowable principle, nor does it make its selection known in any way to a user or metalinguistic student of the language. Surely the idea of a linguistic element this uncontrollably erratic is intolerable. I opt for alternative (5).

Moreover, even if one attempted to shrug off questions about the designata of ϵ-expressions as somehow pseudo-problems, there would still remain (*II*) the little difficulty about the truth-value of a sentence '$G(\epsilon_x Fx)$' when neither this sentence nor its negation is derivable from the accepted postulates of the system. This problem doesn't cause trouble in Hilbertian axiomatics because semantical concepts don't enter there—the focus is on deducibility, and '$G(\epsilon_x Fx)$' may be painlessly consigned to the scrap heap of "undecidable" propositions—undecidable, that is, by formal deduction from the axioms of the system. In application of the ϵ-operator to scientific theories, however, the situation is quite different. A scientist hardly expects his current theories to deductively decide all the questions which interest him. If the theory '$\Phi(t,o)$' has been sufficiently confirmed to be taken seriously and developed further, the obvious next move may be to investigate whether or not it is also the case that $\Psi(t,o)$. But if there is more than one satisfier of '$\Phi(u,o)$', and '$\epsilon_u \Phi(u,o)$' designates just one of these without indicating in any way *which* one, then $\Psi(t,o)$ has a definite truth-value which we have no way to determine unless either $(u)[\Phi(u,o) \supset \Psi(u,o)]$ or $(u)[\Phi(u,o) \supset \sim \Psi(u,o)]$.

In other words, under Carnap's present semantical interpretation of the selection operator, introduction of a theory '$\Phi(t,o)$', where $t = \epsilon_u \Phi(u,o)$, in general introduces a class of sentences containing 't' (or '$\epsilon_u \Phi(u,o)$') *which are factual in the sense of having definite, analytically indeterminate truth-values, yet which are not decidable in any way.*[3] The fault does not lie in the scientific theory, however, but in this construal of the selection-operator. Under (*ii*), above, *any* binding of a (well-formed) sentential function 'Fx' by the ϵ-operator introduces intrinsically undecidable sentences unless for every sentential function 'Gx', either $(x)Gx$ or $(x){\sim}Gx$ or $(\exists x)Fx \cdot (x)(Fx \supset Gx)$ or $(\exists x)Fx \cdot (x)(Fx \supset \sim Gx)$. For example, is

[3]Note that testing the theory '$\Phi(t,o) \cdot \Psi(t,o)$' would not help to determine the truth of '$\Psi(t,o)$' under the theory "$\Phi(t,o)$" since it might very well be that $\epsilon_u \Phi(u,o) \neq [\Phi(u,o) \cdot \Psi(u,o)]$.

or is not $\epsilon_x(x$ is red) squishy? Since the class of red things contains both objects which are squishy (e.g., overripe tomatoes) and ones which are not, there is no way to decide whether $\epsilon_x(x$ is red) is squishy unless the ϵ-operator gives us some clue as to what red object it has singled out as its referent. Or what are the properties of $\epsilon_x(x = x)$? Is it round? Blue? Kind to children? Worth more than \$10 on the open market? It *could* be any of these, and in fact, with some supplementary statistical data, we could even give the probability that it is. Moreover, these questions are *not* meaningless, because if '$\epsilon_x(x = x)$' designates one particular entity e, then by Carnapian semantics the sentence, e.g., '$\epsilon_x(x = x)$ is blue' is true or false according to whether or not e is blue—and both of these semantical alternatives preclude meaninglessness.

So far I have said nothing about the designatum of '$\epsilon_x Fx$' when 'Fx' is unsatisfied. There is nothing in Carnap's Axioms 1 and 2 which necessitates granting a referent to '$\epsilon_x Fx$' in this case, especially in view of increasingly prevalent signs that contemporary philosophers are again coming to countenance the existence of meaningful sentences containing descriptive terms which have no referent. Carnap, however, apparently wishes to arrange for '$\epsilon_x Fx$' to have a designatum no matter what. But not only does the case where 'Fx' is unsatisfied then sustain the same difficulties as those which arise when 'Fx' has more than one satisfier, it compounds the insult by conceding reference to all concepts employed in theories subsequently abandoned as false. What is wrong with a false theory, then, is not that it presupposes entities which don't exist, but that it makes fallacious claims about them. There really is Phlogiston, under this view—it just doesn't behave the way chemists thought it did. Similarly, Pegasus [i.e., $\epsilon_x(x$ is a winged horse captured by Bellerophon)] does, in fact, exist, and for all you know, may be the person sitting across the table from you, or the dust mote you just inhaled.

I, for one, am willing to pay the price of multiple designation and its attendant difficulties in order to grant theoretical concepts—(and also ϵ-expressions, since axiomatic use of the ϵ-operator does not necessitate that the expressions so formed have unique designata, and '$\epsilon_x Fx$' may be semantically explicated as a theoretical term introduced by the theory '$F(\epsilon_x Fx)$',)—genuine referential properties. But could any empiricist conscientiously accede to the cognitive significance of theoretical language at the cost of admitting statements whose truth-values, by their very definition, can never be determined unless special universal quantifications obtain? This is metaphysics with a vengeance; for if we admit—in fact, insist on—the existence of non-analytic, meaningful statements which are in principle immune to empirical test, who amongst us dares to cast the first stone at other epistemic indiscretions.

References

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