V. Causal Systems

"system" concepts are the means by which we cope with causal complexity. Three main themes pervade these; the amalgamation of loci that are causally linked in significant ways, the tracing of causal-propagation lines by iteration of local regularities, and abstractive simplification. In this Section, I shall develop formalisms for catching hold of the first two of these whose full import will not become clear until later. But not to embark upon this Section completely unmotivated, reflect that everyday objects--myself (now), the book (today) that I read yesterday, my malfunctioning pocket calculator (last month), my office furniture and window plants, etc. etc.--are seldom if ever the individual loci of basic causal events. Rather, they are complexes of sub-objects (parts) which are themselves ensembles of sub-parts and so on possibly though not necessarily ad infinitum. And the properties and behavior of any such object or sub-object as a whole are likewise constituted by the properties/behavior of its parts according to finer-grained laws of which the object's molar regularities are abstractive consequences. At some level or levels in this hierarchy of part-analysis, we presumably encounter causal loci whose precursive/excursive/causal properties, and the laws these engage, determine the holistic character of the molar objects they constitute. Seeking to understand the whole in terms of its constituents is essentially what "systems" thinking is all about. The aim of this section is to create a formal framework that effectively deploys the full manifold of diverse conceptual ingredients required for this understanding.

To simplify notation, let us speak of an $\underline{n}$-tuple $\dot{\underline{P}}_1(\underline{a}_1),\ldots,\dot{\underline{P}}_n(\underline{a}_n)$ ($\underline{n} \geq 1$) of events as a single "compound" event $\dot{\underline{P}}(\underline{a})$ in which $\underline{a} = \langle \underline{a}_1,\ldots,\underline{a}_n \rangle$ and $\dot{\underline{P}}(\_\_) = \langle \dot{\underline{P}}_1(\_\_), \ldots, \dot{\underline{P}}_n(\_\_) \rangle$, and say that a dependent event $\dot{\underline{Q}}(\underline{c})$ is caused (in part) by a compound event $\dot{\underline{P}}(\underline{a})$ occurring at "compound" locus $\underline{a}$ just in case $\dot{\underline{Q}}(\underline{c})$ is caused jointly by all component events in $\dot{\underline{P}}(\underline{a})$. We may also understand one compound event to be "contained" in another just in case all component events in the first are also components of the second. Then if the input events in Defs. 2-6 are allowed to be compound, we can without loss of generality take $\underline{n} = 1$ in all form-[8] causal regularities. At times it is convenient to carry this convention even farther. We may define the notion of "compound event" recursively by stipulating that (1) a genuine (primitive, simple, non-compound) event is a compound event of complexity level 0, and (2) for each integer $\underline{n}$, an indexed set of events of complexity level $\underline{n}$ or less, in which at least one is of complexity level $\underline{n}$, is a compound event of complexity level $\underline{n}+1$. (Technically, we would then also want to distinguish among different types of events at complexity level $\underline{n}$ according to the character of the indexing involved.) "Compound locus of complexity level $\underline{n}$" is defined similarly. Then generality-form '$(\forall \underline{x},\underline{z})[\underline{S}(\underline{x},\underline{z}) \supset Q\underline{z} = \underline{f}(P\underline{x})]$' can be taken to subsume very complicated arrays of causal regularities if construed to quantify over compound loci, albeit some care is needed to articulate which components of $\dot{\underline{P}}(\underline{x})$ are causes of which components of $\dot{\underline{Q}}(\underline{z})$. Although it suffices for this Section to presume loci and events only of complexity levels 0 or 1, I shall not make that restriction formally explicit simply because there is no evident gain in doing so.

Ordinary language is exceedingly vague about what logical sorts of entities are to count as "systems." But to be integrated under this term somehow are (1) a set $\underset{\sim}{F}$ of causal regularities, (2) a set $\underset{\sim}{A}$ of entities for which ordinary language has no unambiguous label but in which 'object' (also 'thing') comes close to the mark; (3) a breakdown of each $\underline{A}$ in $\underset{\sim}{A}$ into a collection of locus tuples in the scopes of laws in $\underset{\sim}{F}$; and (4) one or more lag operators that carry some system-objects into others along lines of causal connection. To begin putting these together (though lag will be deferred until later), let us stipulate that for any function-symbol 'ϕ', if the values of ϕ are ordered sets (e.g. tuples) indexed by a common index set $\underline{K}$, 'ϕ_k' designates the function whose value for any argument $\underline{a}$ is the $\underline{k}$th component of $\phi\underline{a}$, i.e. $\phi_k\underline{a} = (\phi\underline{a})_k$. More generally, for any tuple $\bar{\underline{k}} = \langle \underline{k}_1,\ldots,\underline{k}_n \rangle$ of indices in $\underline{K}$, $\phi_{\bar{k}}$ is the function whose value for any argument $\underline{a}$ is tuple $\langle \phi_{k_1}\underline{a},\ldots,\phi_{k_n}\underline{a} \rangle$, and similarly when $\bar{\underline{k}}$ is any other indexed set of indices in $\underline{K}$--i.e. for any ϕ-argument $\underline{a}$, $\phi_{\bar{k}}\underline{a}$ derives from $\bar{\underline{k}}$ by replacing each $\underline{k}$ in $\bar{\underline{k}}$ by $\phi_k\underline{a}$. Then, Definition 5.1. A (locally deterministic) basic causal system (abbreviated "bcs") is a 5-tuple $\Sigma = \langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ such that: (1) $\mathbf{A}$ is a nonempty set, K is a nonempty index set, $\mathbf{F} = \{F_i : k \in K\}$ is a set indexed by K, and ψ and ϕ are both functions on $\mathbf{A}$ whose values are sets each indexed by K. (2) For each k in K, F_k is a causal law $(\forall x,z)[S_k(x,z) \supset Q_k z = f_k(P_k x)]$ in which $\dot{P}_k(x)$ is a compound event (albeit possibly containing only one component event). (3) For each k in K, ψ_k and ϕ_k are functions from $\mathbf{A}$ into F_k's domain and range, respectively, such that $S_k(\psi_k A, \phi_k A)$ for every A in $\mathbf{A}$. (4) For any k and k' in K, $\langle \psi_k A, \phi_k A \rangle = \langle \psi_{k'} A, \phi_{k'} A \rangle$ for some A in $\mathbf{A}$ only if this holds for all A in $\mathbf{A}$, while $\langle F_k, \psi_k, \phi_k \rangle = \langle F_{k'}, \psi_{k'}, \phi_{k'} \rangle$ only if $k = k'$.

If $\langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ is a basic causal system, set $\mathbf{A}$ comprises the system's objects and is the system's molar domain, index set K is the system's organizor, $\langle \psi, \phi \rangle$ is its organization, $\mathbf{F}$ is its inner C(ausal)-structure, and the unordered set of all laws indexed in $\mathbf{F}$ is the system's C(ausal)-character.

To appreciate the sense of Def. 5.1, observe that if $\Sigma = \langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ is a bcs, every Σ-object A in $\mathbf{A}$ corresponds to, though does not explicitly include, an ordered set of facts/events that is not quite but almost fully indexed by K. Specifically, for any object A in the system's molar domain and any index k in its organizor, applying functions ψ_k and ϕ_k (i.e. the kth components of functions ψ and ϕ) to A picks out a particular compound locus $\psi_k A$ and simple locus (or locus tuple) $\phi_k A$. Meanwhile, index k also identifies through Σ's inner C-structure $\mathbf{F}$ a (more or less complex) relational property S_k in which $\psi_k A$ stands to $\phi_k A$, a compound variable P_k whose domain includes $\psi_k A$, and another variable Q_k whose domain includes $\phi_k A$, namely the scope, input variable, and output variable, respectively, of the kth law in $\mathbf{F}$. Together, these coordinate system object A with not only the fact that $S_k(\psi_k A, \phi_k A)$ but also the compound event $\dot{P}_k(\psi_k A)$ (i.e. compound locus $\psi_k A$ having whatever value of compound variable P_k it does have) and the simple event $\dot{Q}_k(\phi_k A)$ (i.e. simple locus or locus tuple $\phi_k A$ having whatever value of simple monadic or relational variable Q_k it does have). These specifications of events $\dot{P}_k(\psi_k A)$ and $\dot{Q}_k(\phi_k A)$ do not identify what values these variables have for these particular loci, anymore than referring to Mary's father by the description 'Mary's father' suffices to identify who he is. But because the one event is caused by the other under law F_k, the value of Q_k for locus $\phi_k A$ is determined in the system by the value of P_k for locus $\psi_k A$. Moreover, it is also possible that some or all of the component events in $\dot{P}_k(\psi_k A)$ are also dependent events "in" A or more precisely coordinated with A by other indices in K. (I.e., there may be one or more k' in K such that $\dot{Q}_{k'}(\phi_{k'} A)$ is a component of $\dot{P}_k(\psi_k A)$.) Thus the total set of events so coordinated with object A in system Σ can be divided into system-input events, mediating events that are determined by causal antecedents acknowledged in the system but in turn help to determine others, and system-output events.

A system's inner <u>C</u>-structure differs from its <u>C</u>-character in that the same law F may be indexed repeatedly in $\mathbf{F}$, corresponding to different locus tuples in F's scope into which $\langle \psi_k, \phi_k \rangle$ maps A for different k in K. Note also that if $\langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ is a <u>bcs</u> then so is $\langle \mathbf{A}', K, \mathbf{F}, \psi', \phi' \rangle$ for any subset A' of A and restriction $\langle \psi', \phi' \rangle$ of $\langle \psi, \phi \rangle$ to $\mathbf{A}'$. It is often heuristically convenient to take a <u>bcs</u>'s molar domain to comprise just a single object.

Evidently, if $\langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ is a basic causal system, then

$$(\forall A)[A \in \mathbf{A} \supset Q_k \phi_k A = f_k(P_k \psi_k A)]$$

for every k in K We may include the K-indexed set of these latter generalities, which quantify over system objects rather than loci, in what we shall later call the system's <u>outer</u> <u>C</u>(<u>ausal</u>)-<u>structure</u>. But a <u>bcs</u> is generally characterized by vastly more regularities than are made explicit by its inner and corresponding outer <u>C</u>-structure. In particular, when a <u>bcs</u> $\langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ is causally recursive, i.e. when some of its events that are outputs under $\mathbf{F}$ are also local inputs for other events whose causation is also made explicit in the system, the system generally has a latent <u>C</u>-structure that includes not merely $\mathbf{F}$ but additional laws derived by composition of laws in $\mathbf{F}$ as well.

[The following paragraphs on law-composition more properly belong in the preceding section on Lawfulness, and will be transferred there in subsequent revisions of this material.]

The composition of one causal law into another is an extremely important concept that is closely related to the mathematics of function-composition but is unhappily rather more complicated than that. Reverting temporally to the more articulate notation of law-form [8], consider two simple causal laws

$$F_1: \quad (\forall x,z)[S_1(x,z) \supset Qz = f_1(Px)] \ ,$$

$$F_2: \quad (\forall x,z)[S_2(x,z) \supset Rz = f_2(Qz)] \ ,$$

and three events $\dot{P}(a)$, $\dot{Q}(b)$, and $\dot{R}(c)$ having respective loci a,b,c such that $S_1(a,b)$ and $S_2(b,c)$. Evidently $\dot{P}(a)$ causes $\dot{Q}(b)$ which in turn causes $\dot{R}(c)$, so by the transitivity of causation $\dot{P}(a)$ causes $\dot{Q}(c)$--but under what law? The functional connection between a's value of P and c's value of R in this case is just $Rc = f_2(f_1(Pa))$, and this relation generalizes to all locus pairs $\langle a',c'\rangle$ for which there is a mediating locus b' appropriately linked under the scope of F_1 to a' on the one hand, and under the scope of F_2 to c' on the other. Thus F_1 and F_2 jointly entail that

$$F_{12}: \quad (\forall x,z)\{(\exists y)[S_1(x,y)\cdot S_2(y,z)] \supset Rz = f_2(f_1(Px))\}$$

is a causal law, under which moreover the causation of $\dot{Q}(c)$ by $\dot{P}(a)$ is subsumed. We want to say that F_{12} is a "composition" of F_1 into F_2 and that this composition is "instantiated" by sequence $\langle a,b\rangle,\langle b,c\rangle$ of locus tuples. However, we also need to generalize these notions to iterated causality governed by laws considerably more complicated than in this extremely simple example. The wanted generalizations are easy enough to intuit; but <u>stating</u> them with technical precision turns out to be surprisingly difficult.

Suppose, for example, that in addition to F_1 and F_2, it is also a law that

$$F_3: \quad (\forall x_1,x_2,z)[S_3(x_1,x_2,z) \supset Rz = f_3(Qx_1,Qx_2)] \ ,$$

where $S_3(x_1,x_2,z)$ entails $x_1 \neq x_2$. (Note that F_2 and F_3 are entirely compatable so long as their scopes are incompatable.) Then F_1 can be composed into both input positions in F_3 to yield causal law

$$F_{113}: (\forall x_1,x_2,z)\{(\exists y_1,y_2)[S_1(x_1,y_1)\cdot S_1(x_2,y_2)\cdot S_3(y_1,y_2,z) \supset Rz = f_3(f_1(Px_1),f_1(Px_2))\}$$

(It is of considerable interest to observe that F_{113} is open regarding whether or not $x_1 = x_2$.) But F_1 can also be composed into just the first input position of F_3, yielding

$$\underline{F}_{13}\text{:}\quad (\forall \underline{x}_1,\underline{x}_2,\underline{z})\,\{(\exists \underline{y})[\underline{S}_1(\underline{x}_1,\underline{y})\cdot \underline{S}_3(\underline{y},\underline{x}_2,\underline{z})] \supset R\underline{z} = \underline{f}_3(\underline{f}_1(P\underline{x}_1),Q\underline{x}_2))\}.$$

So far so good; but if $\underline{F}_2$ is replaced by its logical equivalent

$$\underline{F}_2'\text{:}\quad (\forall \underline{x}_1,\underline{x}_2,\underline{z})[\underline{S}_2'(\underline{x}_1,\underline{x}_2,\underline{z}) \supset R\underline{z} = \underline{f}_2'(P\underline{x}_1) + \underline{f}_2'(P\underline{x}_2)]$$

in which

$$\underline{S}_2'(\underline{x}_1,\underline{x}_2,\underline{z}) =_{def} \underline{S}_2(\underline{x}_1,\underline{z})\cdot(\underline{x}_1 = \underline{x}_2)\ , \qquad \underline{f}_2' =_{def} (1/2)\underline{f}_2\ ,$$

composition of $\underline{F}_1$ into just the first input position of $\underline{F}_2'$ gives

$$\underline{F}_{12}'\text{:}\quad (\forall \underline{x}_1,\underline{x}_2,\underline{z})\{(\exists \underline{y})[\underline{S}_1(\underline{x}_1,\underline{y})\cdot \underline{S}_2'(\underline{y},\underline{x}_2,\underline{z})] \supset R\underline{z} = \underline{f}_2'(\underline{f}_1(P\underline{x}_1)) + \underline{f}_2'(P\underline{x}_2)\},$$

the scope of which is equivalent to $\underline{S}_1(\underline{x}_1,\underline{x}_2)\cdot\underline{S}_2(\underline{x}_2,\underline{z})$. Is $\underline{F}_{12}'$ a causal law? According to HT-8 and Def. 2 (Sect. IV), only if it is proper to view $\dot{\underline{P}}(\underline{a})$ and $\dot{\underline{Q}}(\underline{b})$ as causing $\dot{\underline{R}}(\underline{c})$ jointly when $\underline{S}_1(\underline{a},\underline{b})$ and $\underline{S}_2(\underline{b},\underline{c})$; and since the force of $\underline{F}_1$ and $\underline{F}_2$ (equivalently, $\underline{F}_1$ and $\underline{F}_2'$) is that $\dot{\underline{P}}(\underline{a})$ causes $\dot{\underline{R}}(\underline{c})$ only through mediation by $\dot{\underline{Q}}(\underline{b})$, it seems questionable to claim that $\dot{\underline{P}}(\underline{a})$ works conjointly with $\dot{\underline{Q}}(\underline{b})$ to cause $\dot{\underline{R}}(\underline{c})$. In contrast, for loci $\underline{a},\underline{b}_1,\underline{b}_2,\underline{c}$ such that $\underline{S}_1(\underline{a},\underline{b}_1)$, $\underline{S}_1(\underline{a},\underline{b}_2)$, and $\underline{S}_2(\underline{b}_1,\underline{b}_2,\underline{c})$, so that $\langle\underline{a},\underline{b}_2,\underline{c}\rangle$ is in the scope of $\underline{F}_{13}$ and $R\underline{c} = \underline{f}_3(\underline{f}_1(P\underline{a}),Q(\underline{b}_2))$, it does seem proper to regard $\dot{\underline{P}}(\underline{a})$ and $\dot{\underline{Q}}(\underline{b}_2)$ as joint causes of $\dot{\underline{R}}(\underline{c})$, insomuch as while $\dot{\underline{Q}}(\underline{b}_2)$ mediates one line of influence from $\dot{\underline{P}}(\underline{a})$ to $\dot{\underline{R}}(\underline{c})$, $\dot{\underline{P}}(\underline{a})$ also also affects $\dot{\underline{R}}(\underline{c})$ in an additional way independent of that. And yet it also seems true, under $\underline{F}_{113}$, that there is a sense in which $\dot{\underline{P}}(\underline{a})$ has sole causal responsibility for $\dot{\underline{R}}(\underline{c})$, albeit by way of parallel mediation through $\dot{\underline{Q}}(\underline{b}_1)$ and $\dot{\underline{Q}}(\underline{b}_2)$. (There is nothing genuinely paradoxical about this so long as we acknowledge that the degree of one event's responsibility for another is relative to a specified set of additional causes of the latter; but to our prevailingly still-primitive ways of thinking about causal connection, it certainly seems paradoxical.)

Still, are the intuitions about conjoint causation to which I have just appealed really all that secure? Since nowhere in HT-8, Def. 2, or their discussion have I attempted to clarify what is meant by stipulating in Def. 2 that $\dot{\underline{P}}_1(\underline{x}_1),\ldots,\dot{\underline{P}}_n(\underline{x}_n)$ cause $\dot{\underline{Q}}(\underline{z})$ jointly, we are still very much in need of some analysis of this notion. At one extreme of its possibilities, it might require no more than that $\dot{\underline{P}}_1(\underline{x}_1),\ldots,\dot{\underline{P}}_n(\underline{x}_n)$ are all individually causes of $\dot{\underline{Q}}(\underline{z})$, in which case the present

$\underline{F}'_{12}$ qualifies as a causal law after all. Whereas at another extreme, "joint" causation might involve such an active collaboration among joint causes that in the present example under $\underline{F}_{13}$, $\dot{\underline{P}}(\underline{a})$ and $\dot{\underline{Q}}(\underline{b}_2)$ do not jointly cause $\dot{\underline{R}}(\underline{c})$ when $\underline{S}_1(\underline{a},\underline{b}_1)$ and $\underline{S}_3(\underline{b}_1,\underline{b}_2,\underline{c})$ because $\dot{\underline{P}}(\underline{a})$ does not itself work with $\dot{\underline{Q}}(\underline{b}_2)$ in the requisite fashion even though it brings about another event, $\dot{\underline{Q}}(\underline{b}_1)$, that does. I am not myself able to make sense of a notion of conjoint causation so strong as the latter, and am instead prepared to advance an account of this under which causes $\dot{\underline{P}}_1(\underline{x}_1),\ldots,\dot{\underline{P}}_n(\underline{x}_n)$ of $\dot{\underline{Q}}(\underline{z})$ are also joint causes thereof just in case each one of them matters for $\dot{\underline{Q}}(\underline{z})$ given the remainder. But since that account involves some complications, nor can I show conclusively that it is the correct one, I shall say no more about the nature of conjoint causation here except that this is one more issue of causality where our present understanding is still rudimentary.

What we have just seen is that in attempting to formalize principles of mediated causation, we can generally compose one law into another in more than one way, while moreover the universal conditional that so results may or may not be genuinely causal if the conjointness requirement is at all stringent. For many purposes however, including present efforts to formalize a concept of "causal system" adequate to deep analysis of system structure in its manifold aspects, we can make do quite nicely with a sense of causal law even weaker than that of Def. 2. Specifically, let us say that a generality of form [8] in Def. 2 is a <u>virtual causal law</u> just in case the definiens of Def. 2 obtains after '... are joint causes of ...' is weakened to read '... are each a cause of ...'. In Def. 5.1 and subsequent definitions in this Section, it will be most expedient to take the "causal laws" indexed in <u>bcs</u> Σ's <u>C</u>-structure $\underset{\sim}{\underline{F}}$ to be virtual causal laws. (Even so, should we wish a concept of causal systems in which $\underset{\sim}{\underline{F}}$ is constrained to include only laws that are unqualifiedly causal, or even purely causal in one or another of the stronger senses described by Defs. 3-6, nothing in this Section will require significant modification so long as the same grade of causal law is presumed throughout.) Remaining content with composed laws that are perhaps only virtually causal does not, however, alter the multiplicity of ways to compose one law into another. That is perhaps no great conceptual problem beyond disallowing us to speak of *the* composition of F_i into F_j; but it does make for difficulty in specifying exhaustively what it is for one law to be a composition of others. Let us provisionally try the following:

Definition 5.2. Let F_1, F_2, F_3 be generalities of form [8], i.e., for $j = 1,2,3$, $F_j = (\forall x_1, \ldots, x_{n_j}, z)[S_j(x_1, \ldots, x_{n_j}, z) \supset Q_j z = f_j(P_{j1}x_1, \ldots, P_{jn_j}x_{n_j})]$. Then,

A) F_3 is the *ith-place composition of F_1 into F_2* just in case: (*i*) $Q_1 = P_{2i}$, $Q_3 = Q_2$, and $n_3 = n_1 + n_2 - 1$; (*ii*) $S_3(x_1, \ldots, x_{n_3}, z) =_{def} (\exists y)[S_1(x_1, \ldots, x_{n_1}, y) \cdot S_2(x_{n_1+1}, \ldots, x_{n_1+i-1}, y, x_{n_1+i}, \ldots, x_{n_1+n_2-1}, z)]$. (*iii*) for each input variable P_{3j} ($j = 1, \ldots, n_3$) of F_3, $P_{3j} = P_{2j}$ if $j < i$, $P_{3j} = P_{1(j-i+1)}$ if $i \leq j \leq i - 1 + n_1$, and $P_{3j} = P_{2(j-n_1+1)}$ if $i + n_1 \leq j \leq n_1 + n_2 - 1$; and (*iv*) for any point $\langle P_{21}, \ldots, P_{2(i-1)}, P_{11}, \ldots, P_{1n_1}, P_{2(i+1)}, \ldots, P_{2n_2} \rangle$ in F_3's input space, $f_3(P_{21}, \ldots, P_{2(i-1)}, P_{11}, \ldots, P_{1n_1}, P_{2(i+1)}, \ldots, P_{2n_2}) = f_2(P_{21}, \ldots, P_{2(i-1)}, f_1(P_{11}, \ldots, P_{1n_1}), P_{2(i+1)}, \ldots, P_{2n_2}))$.

B) A tuple $\langle c_1, \ldots, c_{n_c} \rangle$ is the *ith-place condensation* of tuple $\langle a_1, \ldots, a_{n_a} \rangle$ into tuple $\langle b_1, \ldots, b_{n_b} \rangle$ just in case $i \leq n_b$, $a_{n_a} = b_i$, and $\langle c_1, \ldots, c_{n_c} \rangle = \langle b_1, \ldots, b_{i-1}, a_1, \ldots, a_{n_a-1}, b_{i+1}, \ldots, b_{n_b} \rangle$. If F_3 is the *i*th-place composition of F_1 into F_2, a 2-tuple $\langle a, b \rangle$, in which a and b are tuples of loci or locus tuples, *instantiates* the *i*th-place composition of F_1 into F_2 just in case a is in the scope of F_1, b is in the scope of F_2, and some tuple in the scope of F_3 is the *i*th-place condensation of a into b--i.e. just in case $S_1(a)$, $S_2(b)$, and the last component of a is the *i*th component of b.

C) A generality F_{12} is *a simple composition* of F_1 into F_2 just in case, for some integer i, F_{12} is the *i*th-place composition of F_1 into F_2. F_1 is *composable* into F_2 just in case there exists a simple composition of F_1 into F_2.

D) A generality F_n^* of form [8] is an *$(i_2, \ldots, i_n)$-place forward composition* of sequence $F_1, \ldots, F_n$ of form-[8] generalities just in case there exists a generality sequence $F_2^*, \ldots, F_n^*$, terminating in F_n^*, such that F_2^* is the i_2th-place composition of F_1 into F_2 while if $2 < j \leq n$, F_j^* is the i_jth-place composition of F_{j-1}^* into F_j. The $(i_2, \ldots, i_n)$-place forward condensation of tuple-sequence

$\underline{a}_1,\ldots,\underline{a}_n$ is defined similarly. Locus-complex sequence $\underline{a}_1,\ldots,\underline{a}_n$ instantiates $(\underline{i}_2,\ldots,\underline{i}_n)$-place forward composition of $\underline{F}_1,\ldots,\underline{F}_n$ just in case there exists an $(\underline{i}_2,\ldots,\underline{i}_n)$-place forward composition of $\underline{F}_1,\ldots,\underline{F}_n$ while for each $\underline{j} = 2,\ldots,\underline{n}$, $\langle\underline{a}_{j-1},\underline{a}_j\rangle$ instantiates the $\underline{i}_j$th-place composition of $\underline{F}_{j-1}$ into $\underline{F}_j$. Generality-sequence $\underline{F}_1,\ldots,\underline{F}_n$ has a simple forward composition instantiated by $\underline{a}_1,\ldots,\underline{a}_n$ just in case, for some integer tuple $\langle\underline{i}_2,\ldots,\underline{i}_n\rangle$, $\underline{a}_1,\ldots,\underline{a}_n$ instantiates $(\underline{i}_2,\ldots,\underline{i}_n)$-place forward composition of $\underline{F}_1,\ldots,\underline{F}_n$. Sequence $\underline{F}_1,\ldots,\underline{F}_n$ is forwardly composable just in case it has a simple forward composition.

Simple forward compositions of a composable law-sequence $\underline{F}_1,\ldots,\underline{F}_n$ in general comprise only a small proportion of the compositions that can be generated from these laws. (E.g., backward compositions of the sequence, wherein $\underline{F}_{n-1}$ is first composed into $\underline{F}_n$, $\underline{F}_{n-2}$ next composed into the first composition, and so on, include all forward compositions and generally many more as well; while permuting the series before composing forward or backward, or extending it by inserting additional occurrences of laws already included--and it is important to note, e.g., that if $\underline{F}_i$'s output variable is included among its input variables, $\underline{F}_i$ can be composed into itself endlessly--, generally makes still more compositional alternatives available.) But with one major and one minor qualification, all conceivable law-compositions appear to be achievable by iterated simple composition in the sense of Def. 5.2C. The major qualification is that when extending the concept of law-composition to laws containing an infinite number of input variables, we want to be able to compose into infinitely many input components simultaneously--which a finite series of simple compositions cannot achieve. (Fortunately, we shall have no present need for infinite compositions, either simultaneous or successive.[1]) The minor qualification is that if the output variable Q_1 of $\underline{F}_1$ is logically complex, it is possible that some or all components of Q_1 are available to composition in the input of $\underline{F}_2$ even though they have a scattered indexing in $\underline{F}_2$ into which $\underline{F}_1$ cannot be composed under the Def. 5.2B formalism. But so long as the output variable Q of each form-[8] law is constrained to be a 1-tuple, as entailed in Def. 2 by stipulation that for every Q-argument $\underline{c}$, $\dot{\underline{Q}}(\underline{c})$ is a single event, the problem of Q being logically complex does not arise unless a given event can be "single" even while consisting of one or more loci exemplifying a multiplicity of attributes--a prospect that we may take to be analytically incoherent unless no events are so elemental that they cannot be in principle analyzed further. Until we have good reason to work seriously with the prospect of endless analysis, there is perhaps no great urgency about extending Def. 5.2 to accomodate laws with complex output variables.

Def. 5.2 does not stipulate that the form-[8] generalities whose compositions it concerns are necessarily causal laws in any grade of nomicality. But it is important to observe that if $\underline{F}_{12}$ is a simple composition of $\underline{F}_1$ into $\underline{F}_2$, and $\underline{F}_1$ and $\underline{F}_2$ are both virtually causal--as obtains in particular if $\underline{F}_1$ and $\underline{F}_2$ are causal in any of the stronger senses of Defs. 2-6--then $\underline{F}_{12}$ is also a virtual causal law. That is, virtually causal lawfulness is preserved under simple composition. (Under what constraints composition preserves stronger grades of nomicality is unclear.) Moreover, when composition of virtual causal laws is iterated, simple forward compositions of composable law sequences have a special significance in terms of causal chains not in general true of other iterated-composition patterns. Specifically, if $\underline{F}_1,\ldots,\underline{F}_n$ has a simple forward composition instantiated by locus-complex sequence $\underline{a}_1,\ldots,\underline{a}_n$, and for each $\underline{i} = 1,\ldots,\underline{n}$, $\underline{F}_i$ is a virtual causal law while the last component of $\underline{a}_i$ is $\underline{c}_i$ (whence $\underline{c}_i$ is an argument of $\underline{F}_i$'s output variable Q_i), then $\dot{\underline{Q}}_1(\underline{c}_1)\rightarrow\dot{\underline{Q}}_2(\underline{c}_2)\rightarrow\ldots\rightarrow\dot{\underline{Q}}_n(\underline{c}_n)$ is a causal-propagation sequence in which each $\dot{\underline{Q}}_i(\underline{c}_i)$ is a cause of $\dot{\underline{Q}}_{i+1}(\underline{c}_{i+1})$.

Returning, finally, to this Section's concern with formalizing the nature of system structure, we note that whenever, for any indices $\underline{k}$ and $\underline{k}'$ in the organizor of $\underline{bcs}$ Σ, there is a simple composition $\underline{F}^*$ of $\underline{F}_k$ into $\underline{F}_{k'}$ that is instantiated by $\langle\psi_k\underline{A},\phi_k\underline{A}\rangle,\langle\psi_{k'}\underline{A},\phi_{k'}\underline{A}\rangle$ for any Σ-object $\underline{A}$, $\underline{F}^*$ is a law (at least virtually causal) that not only governs events already included in Σ but is entailed to do so, even when $\underline{F}^*$ is not itself indexed in Σ, by Σ's explicit $\underline{C}$-structure $\underset{\sim}{\underline{F}}$ and organization $\langle\psi,\phi\rangle$. Consequently, all such $\underline{F}^*$, and their own compositions instantiated by loci in Σ, etc., may be construed to be part of the system's latent $\underline{C}$-structure. We

By virtue of the transitivity of causal precedence (HT-4), whenever one causal law is composable into another, their composition is also a causal law of at least the domain-extensional grade,[1] albeit the composed law's scope is possibly empty even when this is true of neither composing law. Thus the C-structure of a given bcs can appropriately be construed to include at least latently the compositions of all composable laws that are in its explicit C-character, at least if the system includes some tuples in the compositional scope. We shall formalize this extension later.

So far, Def. 5.1 is little more than an arbitrary formalism, and will largely remain so until Section VII. Still, a sketch of system organization as an omniscient being might develop His conception of a particular instance may be helpful at this point. The role of molar domain $\underset{\sim}{A}$ and functions $\langle \psi, \phi \rangle$ thereon mapping each system object into ordered sets of loci is to replace an ensemble of causally interconnected loci by a single entity of which the former can be construed as parts or attachments. It is possible to do this quite arbitrarily. Thus for any indexed set $\underset{\sim}{F} = \{\underline{F}_k : \underline{k} \in \underline{K}\}$ of causal laws, one almost-trivial way to define a bcs $\langle \underset{\sim}{A}, \underline{K}, \underset{\sim}{F}, \psi, \phi \rangle$ with C-structure $\underset{\sim}{F}$ is to stipulate that $\underset{\sim}{A} = \{\underline{A}_i\}$ is the set of all $\underline{K}$-indexed sets $\underline{A}_i = \{\langle \underline{x}_{ik}, \underline{z}_{ik} \rangle : \underline{k} \in \underline{K}\}$ of ordered pairs such that for each $\underline{k}$ in $\underline{K}$, $\langle \underline{x}_{ik}, \underline{z}_{ik} \rangle$ is in the scope of $\underline{F}_k$ while each ψ_k (ϕ_k) is the function on $\underset{\sim}{A}$ that maps each system object $\underline{A}_i$ into the first (second) component of $\underline{A}_i$'s $\underline{k}$th element $\langle \underline{x}_{ik}, \underline{z}_{ik} \rangle$. But a more instructive way for us to construct, Godlike, a particular bcs whose objects are richly integrated causally, is as follows: Starting with some event $\underline{e}_1$ that interests us, let $\underline{e}_2$ be some part-cause of $\underline{e}_1$. Then by HT-8 there is a law $\underline{F}_1 = (\forall \underline{x}, \underline{z})[\underline{S}_1(\underline{x}, \underline{z}) \supset Q_1(\underline{x}, \underline{z}) = \underline{f}_1(P_1 \underline{x})]$ such that for some locus tuples $\underline{a}$ and $\underline{c}$, $\underline{e}_1 = \dot{\underline{Q}}_1(\underline{c})$ while $\underline{e}_2$ is a component of compound event $\dot{\underline{P}}_1(\underline{a})$. We now build a set $\underset{\sim}{e}$ of events and a set $\underset{\sim}{F}$ of laws recursively, from a base in which $\underset{\sim}{F}$ comprises just $\underline{F}_1$ and $\underset{\sim}{e}$ comprises just $\dot{\underline{Q}}(\underline{c})$ and the events in $\dot{\underline{P}}_1(\underline{a})$, by doing some or all (in any case the third) of the following at each step of the recursion: (1) Add to $\underset{\sim}{e}$ some or all events that are caused by events already in $\underset{\sim}{e}$ under some law already in $\underset{\sim}{F}$. (E.g., the $\dot{\underline{P}}_1(\underline{a})$ already in $\underset{\sim}{e}$ causes not only $\dot{\underline{Q}}(\underline{c})$ but also $\dot{\underline{Q}}(\underline{c}')$ for every $\underline{c}'$ such that $\underline{S}_1(\underline{a}, \underline{c}')$.) (2) For some $\underline{e}_i$ already in $\underset{\sim}{e}$, if $\underline{e}_i'$ is some cause or some effect of $\underset{\sim}{e}_i$ not already in $\underset{\sim}{e}$, add e_i' to $\underset{\sim}{e}$ and to $\underset{\sim}{F}$ add a law under which e_i and e_i' are causally connected if this is not already in $\underset{\sim}{F}$. (3) For every F_k: $(\forall x,z)[S_k(x,z) \supset Q_k z = f_k(P_k x)]$ already in $\underset{\sim}{F}$, and any events $\dot{P}_k(a)$ and $\dot{Q}_k(c)$ such that $S_k(a,c)$, if $\dot{Q}_k(c)$ and some but not all components of $\dot{P}_k(a)$ are already in $\underset{\sim}{e}$, add to $\underset{\sim}{e}$ the other component events in $\dot{P}_k(a)$. Once $\underset{\sim}{e}$ and $\underset{\sim}{F}$ are as comprehensive as we wish, after a finite or infinite number of such steps, let $\underset{\sim}{a}$ be the set of all loci of the events in $\underset{\sim}{e}$, take A to be some fragment of $\underset{\sim}{a}$ sufficient to allow later reconstruction of the whole as wanted, and on the basis of properties and relations that hold in $\underset{\sim}{a}$ other than values of the variables in $\underset{\sim}{F}$, notably ones that constitute the scopes of the laws in $\underset{\sim}{F}$, define a set of relations $\underset{\sim}{\rho}$ such that for every a_i in $\underset{\sim}{a}$, there is at least one ρ_i in $\underset{\sim}{\rho}$ such that A is ρ_i-related to just a_i. From $\underset{\sim}{\rho}$, in turn, it is straightforward to construct an indexing of $\underset{\sim}{F}$, and sets of functions $\{\psi_k\}$ and $\{\phi_k\}$ indexed by the same K that indexes $\underset{\sim}{F}$, such that for each k in K, A is an argument of both ψ_k and ϕ_k while $\langle\psi_k A, \phi_k A\rangle$ is in the scope of F_k. Finally, let $\underset{\sim}{A}$ be the set of all A_i for which the latter conditions remain true when A_i is substituted for A. Then $\langle \underset{\sim}{A}, K, \underset{\sim}{F}, \psi, \phi\rangle$ satisfies Def. 5.1.

The construction just described includes a certain amount of handwaving, most critically in appeal to relations $\underset{\sim}{\rho}$ insomuch as the properties/relations constituting the scopes of the laws in $\underset{\sim}{F}$ may not suffice, for each a_i in $\underset{\sim}{a}$, to relate A to a_i uniquely (though God can presumably find a way). But the example's point is merely to suggest in rough overview how, starting with certain events (more realistically certain kinds of events) that especially interest us, we can develop concepts of <u>objects</u> that are assemblages of loci for events that include not only the ones of focal interest but also the more salient causes and effects of these. Note also--though the point may as yet be a little difficult to appreciate in abstract--how arbitrarily (within limits) we can choose a system's objects. The essential "object" is a set of loci so indexed that its constituents causally coupled by the system's inner C-structure can be picked out by appropriate whole-to-part functions. But any other set of entities that can be put into one-one, or even many-one, correspondence with the set of essential objects will serve just as well. This and related points merit formalization. Let "x is a constituent of y" mean that x is either identical to y, or is an element or indexed component of y, or is an element or indexed component of an element or indexed component of y, etc. Then

<u>Definition 5.3</u>. Let $\Sigma = \langle \underset{\sim}{A}, K, \underset{\sim}{F}, \psi, \phi \rangle$ be a basic causal system while α is a function on $\underset{\sim}{A}$, each value of which is a set of loci indexed by the same set H--i.e., for each A in $\underset{\sim}{A}$ and each h in H, $\alpha_h(A)$ is the locus of a particular causal event. Then α is an <u>L(ocus)-analyzor</u> for Σ just in case: (1) For each A in $\underset{\sim}{A}$ and all h, h' in H, $\alpha_h A$ and $\alpha_{h'} A$ are distinct loci if $h \neq h'$; (2) for each h in H there is at least one (generally many) k in K such that $\alpha_h A$ is a constituent either of $\psi_k A$ or of $\phi_k A$; (3) for each k in K there is a complex $\bar{h}$ of indices in H such that $\langle \psi_k A, \phi_k A \rangle = \alpha_{\bar{h}} A$ for all A in $\underset{\sim}{A}$; and (4) for any A and A' in $\underset{\sim}{A}$, $\alpha A = \alpha A'$ only if $\langle \psi A, \phi A \rangle = \langle \psi A', \phi A' \rangle$. A <u>bcs</u> is <u>L-analyzable</u> just in case it has an L-analyzor. The H-indexed locus set αA into which L-analyzor α of <u>bcs</u> Σ maps any Σ-object A is A's <u>core</u>[2] (relative to α); the set of all Σ-object cores is Σ's <u>core domain</u> (relative to α); and each index $h \in H$ of α is a *<u>locus</u> in Σ (relative to α). [Note that every such *locus h corresponds to a map from Σ-objects into single loci, so that if the molar domain of Σ is restricted to a single object A, *locus h uniquely represents locus $\alpha_h A$.]

Ignoring Clause 4 of Def. 5.3 (which is merely a convenience entailing that there exist functions ψ^* and ϕ^* such that $\psi = \psi^* \alpha$ and $\phi = \phi^* \alpha$, and can always be satisfied by some α when Clauses 1-3 are satisfiable), function α is an L-analyzor for <u>bcs</u> $\langle \underset{\sim}{A}, K, \underset{\sim}{F}, \psi, \phi \rangle$ just in case, for each Σ-object A, αA comprises just the constituents of $\langle \psi A, \phi A \rangle$, indexed by a *locus set H in such fashion that no locus in αA is repeated at different indices, and the system's organization $\langle \psi, \phi \rangle$ picks out locus complexes from the core of each A in A on the basis of those loci's indices in αA.[3] Not all <u>bcss</u> are L-analyzable (e.g. the number of distinct loci in $\langle \psi A, \phi A \rangle$ need not be the same for all A in $\underset{\sim}{A}$); but any can be reduced to one that is just by domain restriction (see Def. 5.4, below). The *locus set for an L-analyzor is arbitrary except for its cardinality, but any two L-analyzors for the same bcs are identical up to a one-one transformation of one's *locus set into the other's.[4] In what follows, I shall usually speak of the L-analyzor, the *locus set, and the core domain for any L-analyzable bcs even though strictly speaking these are unique only up to isomorphism of *loci. Moreover, whenever relevant, I shall also presume any bcs at issue to be L-analyzable.

For any bcs $\Sigma = \langle A, K, F, \psi, \phi \rangle$ with L-analyzor α whose *locus set is H, there exists a unique bcs $\Sigma^* = \langle A^*, K, F, \psi^*, \phi^* \rangle$ with the same organizor K and inner C-structure F as Σ, and having an L-analyzor α^* with the same *locus set H as α, such that A* is the core domain of Σ (i.e. $A^* = \alpha A$), α^* is the Identity function on A^* (i.e. $\alpha^* A^* = A^*$ for each A^* in A^*), and ψ and ϕ are the compositions of α into ψ^* and ϕ^*, respectively (i.e. $\psi = \psi^*\alpha$ and $\phi = \phi^*\alpha$). This Σ^*, which may be called the "core equivalent" or simply the core of Σ, is in effect the ontological essence of Σ. In contrast, a system's molar domain has no inherent ontic significance; for if $\langle A, K, F, \psi, \phi \rangle$ is any L-analyzable bcs, any function β from an arbitrary set B onto A defines an L-analyzable bcs $\langle B, K, F, \psi\beta, \phi\beta \rangle$ with molar domain B but the same core as $\langle A, K, F, \psi, \phi \rangle$. Similarly, two or more bcss can all have the same molar domain A even though each object's core differs from system to system. (Later, I will suggest that this is a large part of why we find it so difficult to make clear the ontology of everyday objects.)

There are several ways in which one bcs can be a set-theoretical fragment of another. Let us say that two bcss $\langle A, K, F, \psi, \phi \rangle$ and $\langle A', K', F', \psi', \phi' \rangle$ are "identical up to indexing" just in case $A = A'$ and for each k in K there is a k' in K', and conversely, such that $F_k = F'_{k'}$, $\psi_k = \psi'_{k'}$, and $\phi_k = \phi'_{k'}$. (When this is so, Clause 4 of Def. 5.1 implies that there is just one k' that corresponds to each k, and conversely, in this way.) Then,

Definition 5.4. Let $\Sigma = \langle A, K, F, \psi, \phi \rangle$ and $\Sigma' = \langle A', K', F', \psi', \phi' \rangle$ be two basic causal systems. Then Σ is an extension of Σ', and Σ' a restriction of Σ, just in case some bcs Σ'' derived from Σ by some not-necessarily-proper restriction of $\underset{\sim}{A}$ and/or $\underline{K}$ is identical with Σ' up to indexing; while the extension or restriction is index-preserving if $\Sigma'' = \Sigma'$. If the restriction is just on $\underset{\sim}{A}$, or just on $\underline{K}$, Σ is respectively a domain extension or content extension of Σ', while Σ' is respectively a domain restriction or content restriction of Σ. If Σ' is a content restriction of Σ that is also index-preserving, i.e. if Σ' derives from Σ just by deleting some indices in the latter's organizor, Σ' is a segment of Σ.

Although the concept of domain extension/restriction is largely trivial, it is important to note that for any nonempty subset $\underset{\sim}{A}'$ of the molar domain $\underset{\sim}{A}$ of any bcs Σ, restriction of Σ to molar domain $\underset{\sim}{A}'$ yields a bcs Σ' that has the same inner $\underline{C}$-structure, same organizor, and (save for domain restriction) same organization as Σ, while every $\underline{A}$ in $\underset{\sim}{A}'$ has the same core in Σ' that it has in Σ. Accordingly, if $\{\underset{\sim}{A}^j\}$ is a partition of molar domain $\underset{\sim}{A}$ such that all objects in each subset $\underset{\sim}{A}^j$ are alike in some respect $\underline{R}$ that interests us, bcs $\Sigma = \langle \underset{\sim}{A}, \underline{K}, \underset{\sim}{F}, \psi, \phi \rangle$ can be viewed as the union of domainwise disjoint but contentwise essentially identical systems $\langle \underset{\sim}{A}^j, \underline{K}, \underset{\sim}{F}, \psi^j, \phi^j \rangle$ each of which has a molar domain $\underset{\sim}{A}^j$ that is homogeneous in respect $\underline{R}$. (Thus in particular, any bcs can be partitioned into contentwise identical systems each of which is $\underline{L}$-analyzable.)

On the other hand, content extensions of a bcs Σ raise points of considerable interest, especially when we consider enlarging just the system's (i) object cores, (ii) its inner $\underline{C}$-structure, or (iii) its organization without changing the system in any other respect beyond the entailed index additions. In Case (i), given bcs $\Sigma = \langle A, \underline{K}, F, \psi, \phi \rangle$ with $\underline{L}$-analyzor α there are in general both trivial and nontrivial ways to enlarge each object's core by adding to it additional loci causally coupled by laws already included in $\underset{\sim}{F}$. Nontrivial core enlargement is illustrated by considering the domain $\underline{B}_k$ and excursor Γ_k of some law $\underline{F}_k$ in $\underset{\sim}{F}$ and adding to each $\alpha\underline{A}$, with appropriate extension of the system in other entailed respects, all not-already-included constituents of every set $\Gamma_k \underline{x}_k$ of loci (or locus tuples if $\underline{F}_k$'s output variable is relational) for which $\underline{x}_k$ is a compound locus in $\underline{B}_k$ whose constituents are all in $\alpha\underline{A}$.

(Since the number of such loci need not be the same for all $\underline{A}$ in $\mathbf{A}$, it may be possible to extend only a domain restriction of Σ in this way.)

Regarding Case (<u>ii</u>), the <u>C</u>-character of a given <u>bcs</u> $\langle \mathbf{A}, \underline{K}', \mathbf{F}', \not\ell', \phi' \rangle$ does not generally include all causal variables on which the system's object-core loci have values. Hence it may be possible to extend the system's organizor from $\underline{K}'$ to $\underline{K}$ (with correlative extensions of $\mathbf{F}'$, $\not\ell'$, and ϕ') so that for each $\underline{k}$ in $\underline{K}$ but not in $\underline{K}'$, $\langle \not\ell_k \underline{A}, \phi_k \underline{A} \rangle$ is in the scope of a law $\underline{F}_k$ not already indexed in $\underline{K}'$ even though all its constituents are already in the core of $\underline{A}$. (Note that the extension in this case does not change any system object's core. Even so, it is again possible that only a domain restriction of the system can be so extended.) Indeed, such extension is generally possible even without introducing any new causal variables, namely by adding to the system's <u>C</u>-character the compositions of laws already therein whose scopes include complexes of loci already in the system's object cores. Carrying law-compositional extension to its limit leads to

<u>Definition 5.5</u>. Basic causal system $\Sigma = \langle \mathbf{A}, \underline{K}, \mathbf{F}, \not\ell, \phi \rangle$ is a <u>law-compositional</u> extension of <u>bcs</u> $\Sigma' = \langle \mathbf{A}', \underline{K}', \mathbf{F}', \not\ell', \phi' \rangle$ just in case the latter is identical up to indexing with an index-preserving content restriction $\Sigma'' = \langle \mathbf{A}, \underline{K}'', \mathbf{F}'', \not\ell'', \phi'' \rangle$ of Σ (whence $\mathbf{A}'' = \mathbf{A}' = \mathbf{A}$ and $\underline{K}'' \subseteq \underline{K}$) such that any <u>L</u>-analyzor for Σ or (if Σ is not <u>L</u>-analyzable) for some domain restriction of Σ is also an <u>L</u>-analyzor for Σ'' or for that same domain restriction of Σ'' (i.e. all loci in each $\underline{A}$ relative to Σ are also in $\underline{A}$ relative to Σ''), while also, for any $\underline{k}$ in $\underline{K}$ but not in $\underline{K}''$, $\underline{F}_k$ is a simple forward composition of a sequence of laws $\underline{F}_{k_1}, \ldots, \underline{F}_{k_n}$ with all $\underline{k}_j$ ($\underline{j} = 1, \ldots, \underline{n}$) in $\underline{K}''$. If moreover for every $\underline{k}$ in $\underline{K}$ but not in $\underline{K}''$, $\underline{F}_k$ is a forward composition of a law-sequence $\underline{F}_{k_1}, \ldots, \underline{F}_{k_n}$ with all $\underline{k}_j$ ($\underline{j} = 1, \ldots, \underline{n}$) in $\underline{K}''$ such that also $\langle \langle \not\ell_{k_1} \underline{A}, \phi_{k_1} \underline{A} \rangle, \ldots, \langle \not\ell_{k_n} \underline{A}, \phi_{k_n} \underline{A} \rangle \rangle$ instantiates this law-composition for ~~some and hence~~ every $\underline{A}$ in $\mathbf{A}$, Σ is an <u>intrinsic</u> law-compositional extension of Σ'. Σ is an <u>extrinsic</u> law-compositional extension of Σ' just in case Σ is a law-compositional extension of Σ' but not an intrinsic one. A <u>bcs</u> is law-compositionally <u>complete</u> just in case it has no proper intrinsic law-compositional

extension, i.e. iff it is identical up to indexing with every one of its intrinsic law-compositional extensions, and is law-compositionally supercomplete just in case it has no proper law-compositional extension either intrinsic or extrinsic. The law-compositional completion of any bcs is any intrinsic law-compositional extension thereof that is law-compositionally complete.

Since ___-is-an-intrinsic-law-compositional-extension-of-___ is transitive and anti-symmetric (treating identity-up-to-indexing as simple identity), it is a partial order on bcss. Hence assuming the Axiom of Choice, it can be shown that every bcs has a law-compositional completion that is unique up to indexing. Note that when F_3 is a composition of laws F_1 and F_2 in Σ's C-character, it is possible that some Σ-object A, some complex a of loci in A's core αA (relative to Σ), and some locus c in A are such that $\langle a, c \rangle$ is in the scope of F_3 even though this is not entailed by any information about which complexes of loci in αA are in the scopes of F_1 and F_2. This is because there may be a locus outside of αA that mediates a causal connection from a to c under F_3, and is the reason why intrinsic law-compositional extensions of a bcs need to be distinguished from extrinsic ones.

Finally, case (iii) of system-content extension acknowledges that Def. 5.1 does not require all complexes of system loci that are in the scopes of the system's laws to be identified by the system's explicit organization. For a given bcs $\Sigma = \langle A, K, F, \psi, \phi \rangle$, it is possible that for some law $F = (\forall x, z)[S(x,z) \supset Qz = f(Px)]$ in the C-character of Σ, and some complexes a and c of loci in the core of some Σ-object A, $\dot{P}(a)$ causes $\dot{Q}(c)$ under F because $S(a,c)$ happens to obtain even though there is no k in K such that $\langle \psi_k A, \phi_k A \rangle = \langle a, c \rangle$ while $F_k = F$. If so, Σ's organization and C structure can be extended (though perhaps only in a domain restriction of Σ) to remedy this lack without changing the system's C character or object cores. Such considerations lead to

Definition 5.6. A bcs $\Sigma = \langle A, K, F, \psi, \phi \rangle$ with L-analyzor α is an organizational extension of bcs $\Sigma' = \langle A', K', F', \psi', \phi' \rangle$ with L-analyzor α' just in case (1) Σ is a content extension of Σ' (whence $A' = A$), (2) each law indexed in F is also indexed in F (whence Σ' has the same C-character as Σ), and (3) for every A in A, all loci in αA are also in $\alpha' A$ (whence αA and $\alpha' A$ are identical up to *locus isomorphism, and α and α' can always be chosen for Σ and Σ', respectively, so that $\alpha' = \alpha$). A bcs is organizationally complete just in case it has no proper organizational extensions, i.e. iff it is identical up to indexing with all of its organizational extensions. The organizational completion of any bcs is any organizational extension thereof that is organizationally complete.

Definition 5.7. A bcs Σ is domain-homogeneous just in case it has an L-analyzor α such that for any given complex h of *loci in α's *locus set, if F is any law in Σ's C-character, $\alpha_{\bar{h}} A$ is in the scope of F for some Σ-object A only if this is so for all Σ-objects.

Every L-analyzable bcs has an organizational completion that is unique up to indexing, and can always be domainwise partitioned into domain-homogeneous systems with the same content as the original. Moreover, if bcs $\langle A, K, F, \psi, \phi \rangle$ is both organizationally complete and domain-homogeneous, a complex $\alpha_{\bar{h}} A$ of loci in the core of any A in A is in the scope of any law F in the system's C-character just in case $\alpha_{\bar{h}} A = \langle \psi_k A, \phi_k A \rangle$ for some k in K. Finally, since the organizational completion of the law-compositional completion (though not generally the converse) of any bcs is both organizationally and law-complete, let us say

Definition 5.8. One bcs is the content closure of another just in case the first is the organizational completion of the law-compositional completion of the second. A bcs is content-closed just in case it is its own content closure.

Obviously, the content closure of any bcs is content-closed. Since any L-analyzor of any bcs Σ is also an L-analyzor of Σ's content closure, and conversely, we can always stipulate this to be the same for both.

A bcs's content closure is its largest content extension that is logically inherent in the original system. Thus when considering the lawfulness of a given bcs, it is appropriate to define its "total C-structure" to comprise both the inner and outer C-structure of its content closure. Beyond that, the system has a "covariational" or "associative" structure comprising all regularities, causal or not, entailed by the system's C-character. (For example, $(\forall \underline{x},\underline{y})[\underline{S}_1(\underline{x},\underline{y}) \supset Q_1\underline{y} = \underline{f}_1(P_1\underline{x})]$ and $(\forall \underline{x},\underline{z})[\underline{S}_2(\underline{x},\underline{z}) \supset Q_2\underline{z} = \underline{f}_2(P_1\underline{x})]$ jointly entail $(\forall \underline{y},\underline{z})[(\exists \underline{x})\{\underline{S}_1(\underline{x},\underline{y}) \cdot \underline{S}_2(\underline{x},\underline{z})\} \supset (\exists \underline{P})\{Q_1\underline{y} = \underline{f}_1(\underline{P}) \cdot Q_2\underline{z} = \underline{f}_2(\underline{P})\}]$, which places constraints on the joint values possible for $Q_1\underline{y}$ and $Q_2\underline{z}$ for any locus pair $\langle \underline{y},\underline{z} \rangle$ in this covariational law's scope. However, we shall have no need to formalize this latter notion.

C-structure or more broadly associative structure, i.e. the causal or causality-derived regularities that constrain a system's causal attributes, is a fundamental kind of system structure, indeed virtually the only kind that is ever acknowledged when behavioral scientists or abstract systems theorists (e.g. Mesarovich & Takahara, 1976) talk about "structure." Yet a second kind, just as fundamental as the first, comprises the extracausal relations among the system's object parts. Let us call this "locus structure," which for the world as a whole, without regard for particular system groupings, might ideally be taken to comprise all logically simple or complex facts whose non-logical ingredients (subjects and attributes) are just loci, precursa, and excursa. (This includes how precursa and excursa are distributed as well as which particular locus tuples instantiate them.) That is, in its purest form, locus structure is what determines which locus tuples lie in the scopes of what causally complete causal laws. In addition, we may also find it convenient to treat formal relations among loci and locus complexes--e.g., whether locus $\underline{z}$ is the $\underline{i}$th component of locus tuple $\underline{x}$, whether locus complexes $\underline{x}$ and $\underline{z}$ have any constituents in common, etc.--to be a variety of locus structure even though it is unclear (see p. 14, above) in what respects if any such relations are ontically "real." (The latter might be called "formal" locus structure in contrast to the first-mentioned "-cursive" variety.) However, this ideal conception of locus structure is neither sharply defined nor usefully applicable to systems whose laws are not stipulated to be pure and causally closed. Pending the appearance of specific needs, therefore, let us provisionally say that the L(ocus)-structure of any given bcs $\Sigma = \langle \underset{\sim}{A}, \underline{K}, \underset{\sim}{F}, \not{v}, \not{o} \rangle$ is, or is characterized by, the $\underline{L}$-analyzor α and organization $\langle \psi^*, \phi^* \rangle$ of Σ's content closure $\Sigma^* = \langle \underline{A}, \underline{K}^*, \underline{F}^*, \psi^*, \phi^* \rangle$, together with the indexed set $\{\underline{S}_k : \underline{k} \in \underline{K}^*\}$ of scopes in $\underline{F}^*$. If Σ^* is also domain-homogeneous (as can always be arranged by suitable domain restriction of Σ), $\langle \psi^*, \phi^* \rangle$ identifies, for each complex $\alpha_{\overline{h}}\underline{A}$ of loci in the core of any Σ-object $\underline{A}$ and each causal law $\underline{F}$ entailed by Σ's $\underline{C}$-character, whether or not $\alpha_{\overline{h}}\underline{A}$ is in the scope $\underline{S}$ of $\underline{F}$ (namely, by whether or not both $\alpha_{\overline{h}}\underline{A} = \langle \psi^*_k\underline{A}, \phi^*_k\underline{A} \rangle$ and $\underline{S} = \underline{S}^*_k$ for some $\underline{k}$ in $\underline{K}^*$), while α identifies for each Σ-object $\underline{A}$ the loci which are its parts or (if $\underline{A}$ is distinct from its core) the appendages through which it participates in the world's causal order. Since we have not required all laws in a bcs's $\underline{C}$-character to be causally pure, nor have we excluded causal attributes from the definitions of $\underline{L}$-analyzors, a particular system's $\underline{L}$-structure is not in general entirely independent of causal attributes. Even so, the system's $\underline{L}$-structure subsumes facts about it that are stipulated in the system's identity, in contrast to the system's conceptually open properties determined by its loci's values on causal variables made explicit as variables in the system's laws.

An awkwardness of this provisional definition of $\underline{L}$-structure is that it is locus-specific and object-specific, i.e. it does not allow us to acknowledge that systems having different core domains, or even differing just in molar objects but not in their cores, may nevertheless be $\underline{L}$-structurally alike. Accordingly, let us say that two bcss Σ and Σ' have the same $\underline{L}$-structure just in case there exists a bcs Σ^* such that Σ and Σ' are each identical up to indexing with some (not necessarily the same) domain restriction of Σ^*. We can also speak of two particular system objects, relative to the bcss of which they are respectively objects, as having or not having the same $\underline{L}$-structure in this sense.

The theory of part-whole and part-part relations in complex systems: Some introductory fragments.

Although I have so far made little effort to make this Section's definitions interpretively meaningful, it will nonetheless be evident that the core of any system object, though conceived as a single entity, is generally an ensemble of loci for causal processes (i.e. cause/effect sequences) that occupy a region of space-time.

Any such object core has a greater or lesser degree of excursive *thickness*, as measured in first-approximation by the maximal length of law-compositional sequences that are instantiated within it. Since causal sequences propagate indefinitely, how we segment their loci into distinct object cores is rather arbitrary. In particular, when for some purposes we find it expedient to highlight system objects with excursively narrow cores (e.g. time slices of temporally enduring Things), we usually find it desirable to keep sight of how these are causally linked as segments of excursively broader objects.

For any *bcs* $\Sigma = \langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$, the $\langle \mathbf{A}, K', \mathbf{F}', \psi', \phi' \rangle$ derived from Σ by restricting K to any nonempty subset K' of K is also a *bcs* and hence a segment of Σ. (This is because the first part of Def. 5.1's Clause 4 preserves under organizor restriction the organizor nonredundancy stipulated by that Clause's second part.) Thus the set of all segments of a given *bcs* Σ is in one-one correspondence with the set of all nonempty subsets of Σ's organizor. For *L*-analyzable *bcss*, however, a more insightful characterization of segments is possible.

Definition 5.9. *Bcs* Σ' is a *full* segment of *bcs* $\Sigma = \langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ just in case Σ' is a segment of Σ such that for every k in the organizor K of Σ, if all constituents of $\langle \psi_k A, \phi_k A \rangle$ are also constituents of $\langle \psi' A, \phi' A \rangle$ for all A in $\mathbf{A}$, where $\langle \psi', \phi' \rangle$ is the organization of Σ', then k is in the organizor K' of Σ'.

Definition 5.10. *Bcs* $\Sigma' = \langle \mathbf{A}, K', \mathbf{F}', \psi', \phi' \rangle$ is a *locus-preserving* segment of *bcs* $\Sigma = \langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ just in case Σ' is a segment of Σ such that for each A in $\mathbf{A}$, all loci in $\langle \psi A, \phi A \rangle$ are also constituents of $\langle \psi' A, \phi' A \rangle$.

For any segment Σ', full or otherwise, of a *bcs* Σ having *L*-analyzor α with *locus set H, the restriction α' of α derived from α by deleting just the *loci in H that index loci in Σ not retained in Σ' is an *L*-analyzor of Σ'. Thus,

Theorem. Let $\Sigma = \langle \mathbf{A}, K, \mathbf{F}, \psi, \phi \rangle$ be a basic causal system having an *L*-analyzor α with *locus set H. Then if *bcs* Σ' is a segment of Σ, Σ' is *L*-analyzed by some *locus restriction α' of α--i.e. α' and α have the same domain $\mathbf{A}$, the *locus set $\underline{H}'$ for α' is a nonempty subset of the *locus set $\underline{H}$ for α, and $\alpha'_h = \alpha_h$ for every $\underline{h}$ in $\underline{H}'$. Conversely, if α' is a *locus restriction of α to *loci in a nonempty subset $\underline{H}'$ of $\underline{H}$, and Σ' is the bcs derived from Σ by restricting the organizor $\underline{K}$ of Σ to the organizor $\underline{K}'$ of Σ' in such fashion that each $\underline{k}$ in $\underline{K}$ is left in $\underline{K}'$ just in case all locus constituents of $\langle \psi_k \underline{A}, \phi_k \underline{A} \rangle$ are in $\alpha'\underline{A}$ for some (equivalently, every) $\underline{A}$ in $\underline{A}$, Σ' is a full segment of Σ for which α' is an $\underline{L}$-analyzor; while any locus-preserving segment of Σ' is also a segment of Σ for which α' is an $\underline{L}$-analyzor.

It follows from this Theorem ~~(which can be taken to define segmentation for bcss that are L-analyzable)~~ that for any bcs $\Sigma = \langle A, \underline{K}, F, \psi, \phi \rangle$ with $\underline{L}$-analyzor α and *locus set $\underline{H}$, the set of all full segments of Σ is in one-one correspondence with the set of all nonempty subsets of $\underline{H}$. Further, the set of all full segments of Σ is in one-one correspondence with the disjoint subsets in a partition of the set of all segments of Σ; specifically, if Σ' is a full segment of Σ, the segments of Σ coordinated with Σ' by the latter correspondence are the locus-preserving segments of Σ'. It is convenient to adopt the notation that if $\underline{t}$ is an index for some subset $\underline{H}_{(t)}$ of $\underline{H}$, $\Sigma_{(t)} = \langle \underline{A}, \underline{K}_{(t)}, \underline{F}_{(t)}, \psi_{(t)}, \phi_{(t)} \rangle$ is some not-necessarily-proper locus-preserving segment of the full segment of Σ corresponding to $\underline{H}_{(t)}$--i.e., $\Sigma_{(t)}$ is an index-preserving content restriction of Σ whose object cores are restricted exactly to loci with indices in $\underline{H}_{(t)}$--while $\alpha_{(t)}$ is the $\underline{L}$-analyzor for $\Sigma_{(t)}$ derived from α by restriction of $\underline{H}$ to $\underline{H}_{(t)}$. Using this notation, we may then say

Definition 5.11. An indexed set $\Sigma^2_T = \{\Sigma_{(t)} : \underline{t} \in \underline{T}\}$ of basic causal systems is a segmented causal system ("scs") just in case it is a $\underline{T}$-indexed set of segments of some $\underline{L}$-analyzable basic causal system, i.e. just in case there exists a bcs $\Sigma = \langle \underline{A}, \underline{K}, \underline{F}, \psi, \phi \rangle$ having an $\underline{L}$-analyzor α with *locus set $\underline{H}$, such that (1) $\underline{T}$ also indexes a set $\{\underline{H}_{(t)} : \underline{t} \in \underline{T}\}$ of nonempty subsets of $\underline{H}$, and (2) for each $\underline{t}$ in $\underline{T}$, $\Sigma_{(t)} = \langle \underline{A}, \underline{K}_{(t)}, \underline{F}_{(t)}, \psi_{(t)}, \phi_{(t)} \rangle$ is a segment of Σ for which the restriction $\alpha_{(t)}$ of α to *loci in $\underline{H}_{(t)}$ is an $\underline{L}$-analyzor. Any bcs Σ so related to scs Σ^2_T is by definition a generator of Σ^2_T.

Note that Def. 5.11 does not require subsets $\{\underline{H}_{(t)}\}$ of $\underline{H}$ to be disjoint (we shall, in fact, have important use to make of this permitted overlap). Neither does the definition require the union of $\{\underline{H}_{(t)}\}$ over all $\underline{t}$ in $\underline{T}$ to exhaust $\underline{H}$, which is one of the reasons why a given scs has many different generators. Every scs does have a unique minimal generator, however:

Definition 5.12. Bcs Σ is respectively the union or intersection of a segmented causal system Σ_T^2 just in case each $\Sigma_{(t)}$ in Σ_T^2 is a segment of Σ, or Σ is a segment of each $\Sigma_{(t)}$, and the organizor $\underline{K}$ of Σ is the union, or intersection, over $\underline{T}$ of the organizors $\{\underline{K}_{(t)}\}$ of $\{\Sigma_{(t)}\}$.

It is then easily seen that

Theorem. Any segmented causal system Σ_T^2 has exactly one union $U\Sigma_T^2$, which is both a generator of Σ_T^2 and a segment of any other generator of Σ_T^2. Σ_T^2 has at most one intersection which, if it exists, is the union of all Σ' such that Σ' is a segment of every $\Sigma_{(t)}$ in Σ_T^2.

The union of scs Σ_T^2 is not in general identical with some other generator Σ^* of Σ_T^2, not even when $U\Sigma_T^2$ is organizationally complete and Σ^* is only a content extension of $U\Sigma_T^2$, because Σ^*'s $\underline{C}$-character may contain laws that are compositions of laws contained in the union of the $\underline{C}$-characters of the $\Sigma_{(t)}$, perhaps even in the $\underline{C}$-character of just one of them, when the composition is too global (i.e. excursively extended) to be instantiated in any one segment $\Sigma_{(t)}$. In that case, it may or may not be possible to reclaim the organization of Σ^* not made explicit in $U\Sigma_T^2$ by some intrinsic law-compositional extension of the latter. In light of that possibility, let us say

Definition 5.13. Segmented causal system Σ_T^2 segmentizes (equivalently, is a segmentation of) basic causal system Σ just in case each $\Sigma_{(t)}$ in Σ_T^2 is a segment of Σ and Σ is a not-necessarily-proper segment of the law-compositional closure of the union of Σ_T^2. If each $\Sigma_{(t)}$ in Σ_T^2 is a full segment of Σ, Σ_T^2 is a full segmentation of Σ.

A segmentation of Σ is in effect a dissection of Σ into generally-overlapping parts in such fashion that these parts collectively retain enough of the original to allow reconstitution of Σ as a whole. As already noted in slightly different terms, we cannot produce a segmentation of Σ just by collecting a set of segments of Σ that jointly contain all loci in Σ ~~in fact, an arbitrary scs does not even necessarily segmentize its union~~ so segmentizing a given <u>bcs</u> Σ is not a trivial exercise. Even less trivial is to find segmentations of Σ that parse Σ's total structure in ways that are <u>insightful</u>. Just what these ways may be is an open-ended question to which we have scarcely begun to articulate any comprehensive answers. But fragments of the story are prominent in traditional systems thinking, two of which in particular are so foundational that it is important to introduce them here even though a proper development of them is neither practical nor necessary on this occasion.

For one, it is very possible that the total structure of a given <u>bcs</u> Σ is built out of a small number of simple structures that recur repeatedly throughout a segmentation of Σ. We can formalize this notion as follows:

<u>Definition 5.14</u>. <u>Bcs</u> Σ^* is a <u>structural prototype</u> for <u>bcs</u> $\Sigma = \langle A, K, F, \psi, \phi \rangle$ under domain transformation β just in case β is a one-one function from A onto a set B such that <u>bcs</u> $\langle B, K, F, \psi\beta^{-1}, \phi\beta^{-1} \rangle$ is identical up to indexing with a not-necessarily-proper domain restriction of Σ^*.

<u>Theorem</u>. If <u>bcs</u> Σ^* is a structural prototype for <u>bcs</u> Σ under domain transformation β, and α^* is an <u>L</u>-analyzor for Σ^*, then α^*/β (i.e. the composition of β into α^*) is an <u>L</u>-analyzor for Σ.

<u>Definition 5.15</u>. Two <u>bcss</u> Σ and Σ' have the <u>same structure</u> (both <u>L</u>-structure and <u>C</u>-structure) just in case there exist domain transformations for Σ and Σ' respectively under which some Σ^* is a structural prototype for both Σ and Σ'. [Note: this is an equivalence relation on <u>bcss</u>.]

Definition 5.16. For any scs Σ_T^2 indexed by T and having molar domain A (i.e. A is the molar domain of each $\Sigma_{(t)}$ in Σ_T^2), let $\beta_{(t)}$ for each t in T be the function on A such that $\beta_{(t)}A = \langle A, t\rangle$ for each A in A. Then a structural analysis of scs Σ_T^2 is a pair $\langle S, \sigma\rangle$ in which S is a set of L-analyzable bcss no two of which have the same structure and σ is a function from T into S such that for each t in T, σt is a structural prototype for $\Sigma_{(t)}$ under domain transformation $\beta_{(t)}$.

Every scs Σ_T^2 has a structural analysis $\langle S, \sigma\rangle$ that can be developed by first transforming the molar domain of each $\Sigma_{(t)}$ in Σ_T^2 from A to $A_{(t)} = \beta_{(t)}A$; sorting the domain-transformed $\{\Sigma_{(t)}\}$ into same-structure equivalence classes; and combining (by molar-domain union) the segments in each equivalence class, after these are also re-indexed to have the same organizor, to form the bcs in S that is the structural prototype for each segment in this equivalence class. Note further that if $\langle S, \sigma\rangle$ is a structural analysis of scs Σ_T^2 whose union $U\Sigma_T^2 = \langle A, K, F, \psi, \phi\rangle$ has L-analyzor α with *locus set H, then if $\sigma t = \sigma t'$ for any two segments $\Sigma_{(t)}$ and $\Sigma_{(t')}$ in Σ_T^2, $\langle S, \sigma\rangle$ determines a one-one correspondence between *loci in $H_{(t)}$ and *loci in $H_{(t')}$ such that if $\bar{h}$ is any complex of *loci in $H_{(t)}$ and $\bar{h}'$ is the corresponding complex of *loci in $H_{(t')}$, then for any molar objects A and A' in A, and any k in K, the complex $\alpha_{(t)\bar{h}}A$ of loci in the core of A under $\Sigma_{(t)}$ is in the scope of law F_k just in case the complex $\alpha_{(t')\bar{h}'}A'$ of loci in the core of A' under $\Sigma_{(t')}$ is also in F_k's scope.

If $\langle S, \sigma\rangle$ is a structural analysis of Σ_T^2, $\sigma t = \sigma t'$ for any t and t' in T just in case $\Sigma_{(t)}$ and $\Sigma_{(t')}$ have the same structure. Other things equal, it seems advantageous to segmentize a given bcs Σ in such fashion that a maximal number of these segments exemplify a minimum number of different structures. However, segmentizing Σ to display a high degree of modular repetitiousness in this way is best viewed as secondary to (though generally co-pursuable with) segmentizing Σ to parse its causal evolution. In real-life applications of systems thinking, we find ourselves highly motivated to analyze each enduring Thing (i.e., molar object in an excursively extended system) into temporal "stages" that partake of a before/after ordering through which causation propagates. To capture this intuition comprehensively, with sufficient technical depth and flexibility to subsume traditional models of system dynamics while exercising a capacity to generalize insightfully beyond the restrictive special presuppositions of these, is perhaps the most important goal to which a theory of system structure can aspire.

Definition 5.17. Let $\Sigma = \langle \underset{\sim}{A}, K, F, \psi, \phi \rangle$ be a basic causal system and recall that for each k in K, F_k is a causal law $(\forall x, z)[S_k(x,z) \supset Q_k z = f_k(P_k x)]$. Then for any Σ-object A and any event e: (1) e is antecedent in (the core of) A under Σ just in case e is a component of compound event $\dot{P}_k(\psi_k A)$ for some k in K. (2) e is consequent in A (under Σ) just in case $e = \dot{Q}_k(\phi_k A)$ for some k in K. (3) e is an event of A (under Σ) just in case e is either antecedent or consequent in A under Σ. (4) e is an input (output) event of A under Σ just in case e is antecedent but not consequent (consequent but not antecedent) in A under Σ. (5) e is a mediation event of A under Σ just in case e is both antecedent and consequent in A under Σ.

Definition 5.18. Bcs Σ is externally determined just in case it has an intrinsic law-compositional extension $\Sigma^* = \langle \underset{\sim}{A}, K^*, F^*, \psi^*, \phi^* \rangle$ such that for every k in K^*, there is a k' in K^* (not always $k' \neq k$) for which $Q^*_{k'} = Q^*_k$, $\phi^*_{k'} = \phi^*_k$, and every component event in compound event $\dot{P}^*_{k'}(\psi^*_{k'}A)$ for any A in $\underset{\sim}{A}$ is an input event in A under Σ^*.

Bcs Σ is externally determined just in case any consequent event of any Σ-object A is determined just by input events of A under a finite sequence of Σ's laws by applications made explicit in Σ. Any bcs is externally determined if its organizor has finite cardinality. On the other hand, it is entirely possible for a bcs with infinite organizor to contain infinite causal precessions, each event in which having at least one cause also within the precession. (Systems that are not externally determined raise some *very* interesting questions--scientific, philosophical, and mathematical--that must be passed by here.)

Definition 5.19. Let $\underline{k}_1,\ldots,\underline{k}_n$ ($\underline{n} \geq 2$) be a finite sequence of indices in the organizor $\underline{K}$ of bcs $\Sigma = \langle \underline{A}, \underline{K}, \underline{F}, \psi, \phi \rangle$. Then $\underline{k}_1,\ldots,\underline{k}_n$ is a C(ausal)-chain in Σ just in case law sequence $\underline{F}_{k_1},\ldots,\underline{F}_{k_n}$ has a simple forward composition instantiated by $\langle \psi_{k_1}\underline{A}, \phi_{k_1}\underline{A} \rangle,\ldots,\langle \psi_{k_n}\underline{A}, \phi_{k_n}\underline{A} \rangle$ for some (or equivalently, if Σ is L-analyzable, every) Σ-object $\underline{A}$. For any two indices $\underline{k}$ and $\underline{k}'$ in $\underline{K}$, $\underline{k}$ Σ-precedes $\underline{k}'$ just in case there is a C-chain $\underline{k}_1,\ldots,\underline{k}_n$ in Σ such that $\underline{k}_1 = \underline{k}$ and $\underline{k}_n = \underline{k}'$, while $\underline{k}$ Σ-matches $\underline{k}'$ just in case $\langle Q_k, \phi_k \rangle = \langle Q_{k'}, \phi_{k'} \rangle$ (where as before Q_k is the output variable of $\underline{F}_k$), and $\underline{k}$ Σ-forecasts $\underline{k}'$ just in case $\underline{k}$ Σ-precedes some $\underline{k}''$ in $\underline{K}$ that Σ-matches $\underline{k}'$.

Theorem. Σ-match is an equivalence relation (i.e., reflexive, symmetric, and transitive) on indices in Σ's organizor.

Theorem. If $\underline{k},\underline{k}_1,\ldots,\underline{k}_n,\underline{k}'$ is a C-chain in Σ, so are $\underline{k},\underline{k}_1,\ldots,\underline{k}_n$ and $\underline{k}_1,\ldots,\underline{k}_n,\underline{k}'$. (I.e., any consecutive subsequence of a C-chain in Σ is also a C-chain in Σ.)

Theorem. If $\underline{k}$ Σ-precedes $\underline{k}'$, then $\underline{k}$ Σ-forecasts $\underline{k}'$. If $\underline{k}$ Σ-matches $\underline{k}'$ and $\underline{k}'$ Σ-precedes $\underline{k}''$, then $\underline{k}$ Σ-precedes $\underline{k}''$. If $\underline{k}$ Σ-forecasts $\underline{k}'$ and $\underline{k}'$ either Σ-precedes or Σ-forecasts $\underline{k}''$, then $\underline{k}$ respectively either Σ-precedes or Σ-forecasts $\underline{k}''$. Corollary: The relations of Σ-precedence and Σ-forecast are both transitive.

If $\underline{k}$ Σ-precedes $\underline{k}'$ and Σ is L-analyzable, then for every Σ-object $\underline{A}$, event $\dot{\underline{Q}}_k(\phi_k\underline{A})$ as well as well as each event in $\dot{\underline{P}}_k(\psi_k\underline{A})$ is a cause of event $\dot{\underline{Q}}_{k'}(\phi_{k'}\underline{A})$ in a fashion made explicit by covering laws and mediating events identified in Σ. (If $\underline{k}'$ immediately follows $\underline{k}$ in all C-chains from $\underline{k}$ to $\underline{k}'$, no mediating events are acknowledged in Σ between $\dot{\underline{Q}}_k(\phi_k\underline{A})$ and $\underline{Q}_{k'}(\phi_{k'}\underline{A})$.) The same is true if $\underline{k}$ Σ-forecasts $\underline{k}'$, though if $\underline{k}$ merely Σ-forecasts $\underline{k}'$ without Σ-preceding it, the causes of $\dot{\underline{Q}}_{k'}(\phi_{k'}\underline{A})$ made explicit in $\dot{\underline{P}}_{k'}(\psi_{k'}\underline{A})$ may not include any of the events mediating between $\dot{\underline{Q}}_k(\phi_k\underline{A})$ and $\dot{\underline{Q}}_{k'}(\phi_{k'}\underline{A})$. If $\underline{k}$ Σ-matches $\underline{k}'$, $\dot{\underline{Q}}_k(\phi_k\underline{A})$ and $\dot{\underline{Q}}_{k'}(\phi_{k'}\underline{A})$ are the same event, albeit the causes thereof made explicit in Σ by k (namely, the component events in $\dot{P}_k(\psi_k A)$) may not be the same as the ones made explicit by k'.

Since Σ-precedence is transitive, and by postulation (cf. HT-4) is also antisymmetric and irreflexive, Σ-precedence is a strict partial order on Σ's organizor indices. So is Σ-forecast. Indeed, Σ-forecast is almost the same relation as Σ-precedence; the only difference is that when k Σ-precedes k', k may not Σ-precede all k" that Σ-match k'.

Definition 5.20. Let $\Sigma_{(s)}$ and $\Sigma_{(t)}$ be any two segments of bcs Σ. Then $\Sigma_{(s)}$ is a Σ-ancestor of $\Sigma_{(t)}$ just in case (1) every index in $\Sigma_{(s)}$'s organizor $K_{(s)}$ Σ-matches or Σ-forecasts some index in $\Sigma_{(t)}$'s organizor $K_{(t)}$; (2) every index in $K_{(t)}$ that Σ-matches some index in $\Sigma_{(s)}$ is also in $\Sigma_{(s)}$; and (3) for every C-chain $k_1,\ldots,k_n$ in Σ, if k_1 is in $K_{(t)}$ while k_n Σ-matches some index in $K_{(s)}$ then all of $k_1,\ldots,k_n$ are in both $K_{(s)}$ and $K_{(t)}$. [Note: Clause 2 is conceptually a limiting case of Clause 3.] A segment $\Sigma_{(t)}$ of Σ is Σ-compact just in case $\Sigma_{(t)}$ is a Σ-ancestor of itself. $\Sigma_{(s)}$ is Σ-immanent for $\Sigma_{(t)}$ just in case $\Sigma_{(s)}$ is a Σ-ancestor of $\Sigma_{(t)}$ and the union of $\Sigma_{(s)}$ and $\Sigma_{(t)}$ is Σ-compact.

To avoid the awkwardness of saying that event e is a particular kind of event of A under one Σ-segment $\Sigma_{(t)}$ but perhaps not under another (since A has different cores under different Σ-segments), let us refer to events of $A_{(t)}$ (where $A_{(t)} = \beta_{(t)}A = \langle A,t\rangle$ as defined previously) rather than of A under $\Sigma_{(t)}$. (Events of A's core under $\Sigma_{(t)}$ are identical with events of the core of $A_{(t)}$ under the bcs resulting from the $\beta_{(t)}$-transformation of $\Sigma_{(t)}$'s molar domain, so this shift of terminology preserves the sense of Def. 5.17 while obviating the need for explicit mention of the Σ-segment under which e is or is not an event of $A_{(t)}$.) Then for one segment $\Sigma_{(s)}$ of global bcs Σ to be a "Σ-ancestor" of another Σ-segment $\Sigma_{(t)}$, Def. 5.20 requires that for each Σ-object A, every event e of $A_{(s)}$ is either a Σ-explicit cause of some event of $A_{(t)}$ or is itself an event of $A_{(t)}$: whereas in contrast, no causal sequence $\underline{e}_1 \rightarrow \underline{e}_2 \rightarrow \ldots \rightarrow \underline{e}_n$ progresses from a consequent event $\underline{e}_1$ of $\underline{A}_{(t)}$ to an event $\underline{e}_n$ of $\underline{A}_{(s)}$, with the causation of each $\underline{e}_{i+1}$ by $\underline{e}_i$ made explicit by Σ, unless $\underline{e}_1 \rightarrow \underline{e}_2 \rightarrow \ldots \rightarrow \underline{e}_n$ is a causal sequence of events common to both $\underline{A}_{(s)}$ and $\underline{A}_{(t)}$ with the causation made explicit for $\underline{A}_{(s)}$ by $\Sigma_{(s)}$ and for $\underline{A}_{(t)}$ by $\Sigma_{(t)}$. For $\Sigma_{(t)}$ to be "Σ-compact," if $\underline{e}_1 \rightarrow \underline{e}_2 \rightarrow \ldots \rightarrow \underline{e}_n$ is a Σ-explicit causal sequence in which $\underline{e}_1$ and $\underline{e}_n$ are both events of $\underline{A}_{(t)}$ (more precisely with $\underline{e}_1$ consequent in $\underline{A}_{(t)}$), all events in this sequence must be events of $\underline{A}_{(t)}$ with the causation made explicit by $\Sigma_{(t)}$. Finally, for a Σ-ancestor $\Sigma_{(s)}$ of $\Sigma_{(t)}$ to be "Σ-immanent" for $\Sigma_{(t)}$, both $\Sigma_{(s)}$ and $\Sigma_{(t)}$ must be Σ-compact while moreover, for any Σ-explicit causal sequence $\underline{e}_1 \rightarrow \ldots \rightarrow \underline{e}_n$ that passes from an event $\underline{e}_1$ of $\underline{A}_{(s)}$ but not of $\underline{A}_{(t)}$ to an event $\underline{e}_n$ of $\underline{A}_{(t)}$ but not of $\underline{A}_{(s)}$ must be mediated by one or more consecutive events $\underline{e}_m \rightarrow \ldots \rightarrow \underline{e}_{m'}$, $(1 < \underline{m} \leq \underline{m}' < \underline{n})$ in this sequence that are events of both $\underline{A}_{(s)}$ and $\underline{A}_{(t)}$. That is, when $\Sigma_{(s)}$ is Σ-immanent for $\Sigma_{(t)}$, all Σ-explicit causation from events of $\underline{A}_{(s)}$ to events of $\underline{A}_{(t)}$ passes through an interface between $\underline{A}_{(s)}$ and $\underline{A}_{(t)}$ comprising events common to both. To put the point still another way, when $\Sigma_{(s)}$ is Σ-immanent for $\Sigma_{(t)}$, determination of one event by others <u>within</u> both $\underline{A}_{(s)}$ and $\underline{A}_{(t)}$ separately is nomic; but determination passes from one to the other by the logical connection of identity.

By virtue of the properties of $\underline{C}$-chains and Σ-forecast already noted, it is elementary (but very much nontrivial) to see that

<u>Theorem</u>. Let $\Sigma_{(r)}$, $\Sigma_{(s)}$, and $\Sigma_{(t)}$ be any segments of <u>bcs</u> Σ. Then: (<u>a</u>) If $\Sigma_{(r)}$ is a Σ-ancestor of $\Sigma_{(s)}$ and $\Sigma_{(s)}$ is a Σ-ancestor of $\Sigma_{(t)}$, then $\Sigma_{(r)}$ is a Σ-ancestor of $\Sigma_{(t)}$. (<u>b</u>) If $\Sigma_{(s)}$ and $\Sigma_{(t)}$ are Σ-ancestors of each other, then $\Sigma_{(s)} = \Sigma_{(t)}$.[5] (<u>Corollary</u>: Σ-ancestory is transitive and anti-symmetric, and is hence a partial order on Σ-segments.) (<u>c</u>) If $\Sigma_{(r)}$ and $\Sigma_{(s)}$ are both Σ-ancestors of $\Sigma_{(t)}$, any segment of the union of $\Sigma_{(r)}$ and $\Sigma_{(s)}$ is a Σ-ancestor of $\Sigma_{(t)}$. (<u>d</u>) If $\Sigma_{(r)}$ is a Σ-ancestor of both $\Sigma_{(s)}$ and $\Sigma_{(t)}$, it is also a Σ-ancestor of their union. (<u>e</u>) If $\Sigma_{(s)}$ is Σ-immanent for $\Sigma_{(t)}$, $\Sigma_{(s)}$ and $\Sigma_{(t)}$ are both Σ-compact. (<u>f</u>) If $\Sigma_{(r)}$ is Σ-immanent for $\Sigma_{(s)}$ and $\Sigma_{(s)}$ is Σ-immanent for $\Sigma_{(t)}$, $\Sigma_{(r)}$ is

Σ-immanent for the union of $\Sigma_{(s)}$ and $\Sigma_{(t)}$, and the union of $\Sigma_{(r)}$ and $\Sigma_{(s)}$ is Σ-immanent for $\Sigma_{(t)}$.

Clause 1 of this definition of Σ-ancestory is a rather strong condition that one might well wish to relax. (That is, shouldn't we allow one Σ-segment to be a causal predecessor of another without requiring every event in the first to have some causal effect on the second?) However, the wanted relaxation is difficult to bring off while preserving the relation as a partial order. Instead, it seems more advantageous to retain Σ-ancestory in its strong Def. 5.20 sense for our basic causal ordering of Σ-segments, but to augment Σ-compact Σ-segments partaking of a Σ-ancestory structure by certain auxillary Σ-segments distinctively appropriate to the former.

Definition 5.21. A segment $\Sigma_{(t)}$ of bcs Σ is Σ-terminal just in case for all $\underline{k}$ and $\underline{k}'$ in the organizor $\underline{K}$ of Σ, if $\underline{k}$ Σ-forecasts $\underline{k}'$ and $\underline{k}$ is in the organizor $\underline{K}_{(t)}$ of $\Sigma_{(t)}$, then $\underline{k}'$ is also in $\underline{K}_{(t)}$. $\Sigma_{(t)}$ is Σ-initial just in case for all $\underline{k}$ and $\underline{k}'$ in $\underline{K}$, if $\underline{k}$ Σ-forecasts $\underline{k}'$ and $\underline{k}'$ is in $\underline{K}_{(t)}$, then $\underline{k}$ is also in $\underline{K}_{(t)}$.

Theorem. If $\Sigma_{(t)}$ is a segment of Σ while $\Sigma_{(t')}$ is a segment of $\Sigma_{(t)}$, then $\Sigma_{(t')}$ is Σ-terminal (Σ-initial) if and only if it is $\Sigma_{(t)}$-terminal ($\Sigma_{(t)}$-initial).

Theorem. If Σ-segments $\Sigma_{(s)}$ and $\Sigma_{(t)}$ are both Σ-terminal (Σ-initial) then the union and, if it exists, the intersection of $\Sigma_{(s)}$ and $\Sigma_{(t)}$ are both Σ-terminal (Σ-initial).

If $\Sigma_{(t)}$ is Σ-terminal, all Σ-explicit causal progressions issuing from a consequent event of $\underline{A}_{(t)}$ remain in $\underline{A}_{(t)}$. If $\Sigma_{(t)}$ is Σ-initial, all Σ-explicit causal progressions arrive at an event of $\underline{A}_{(t)}$ only from other events of $\underline{A}_{(t)}$.

Definition 5.22. Let $\Sigma_{(t)}$ be a segment of bcs Σ. Then a segment $\Sigma_{(t'')}$ of $\Sigma_{(t)}$ is the output section of $\Sigma_{(t)}$ (relative to Σ) just in case $\Sigma_{(t'')}$ is the union of all Σ-initial segments of $\Sigma_{(t)}$. $\Sigma_{(t')}$ is the input section of $\Sigma_{(t)}$

(relative to Σ) just in case $\Sigma_{(t')}$ is the union of all Σ-initial segments of $\Sigma_{(t)}$. $\Sigma_{(t*)}$ is the <u>state section</u> of $\Sigma_{(t)}$ (relative to Σ) just in case $\Sigma_{(t*)}$ is the union of all segments of $\Sigma_{(t)}$ that are disjoint from both the input section and the output section of $\Sigma_{(t)}$. [Note: Any one or two of these three sections may not exist for a given $\Sigma_{(t)}$. Also, the input and output sections of $\Sigma_{(t)}$ need not be disjoint.]

When $\Sigma_{(t')}/\Sigma_{(t*)}/\Sigma_{(t'')}$ are respectively the input/state/output sections of $\Sigma_{(t)}$, for each Σ-object $\underline{A}$ we may correspondingly speak of $\underline{A}_{(t')}/\underline{A}_{(t*)}/\underline{A}_{(t'')}$ as the input/state/output sections of $A_{(t)}$.

<u>Theorem</u>. A Σ-organizor index $\underline{k}$ is in the organizor of the state section of Σ-segment $\Sigma_{(t)}$ just in case there exist Σ-organizor indices $\underline{k}'$ and $\underline{k}''$ such that $\underline{k}'$ Σ-forecasts $\underline{k}$ which in turn Σ-forecasts $\underline{k}'$, while neither $\underline{k}'$ nor $\underline{k}''$ is in the organizor of $\Sigma_{(t)}$.

That is, every event in the state section of $\underline{A}_{(t)}$ has both Σ-explicit causes and Σ-explicit effects outside of $\underline{A}_{(t)}$, whereas this is not true of any events in either the input or the output sections of $\underline{A}(t)$.

<u>Definition 5.23</u>. Let $\Sigma_{(s)}$ and $\Sigma_{(t)}$ be any segments of <u>bcs</u> Σ. Then $\Sigma_{(s)}$ and $\Sigma_{(t)}$ are <u>statewise equivalent</u> (relative to Σ) just in case $\Sigma_{(s)}$ and $\Sigma_{(t)}$ have the same state section (relative to Σ). $\Sigma_{(s)}$ is a <u>state-ancestor</u> of $\Sigma_{(t)}$ (relative to Σ) just in case the state section of $\Sigma_{(s)}$ (relative to Σ) is a Σ-ancestor of the state section of $\Sigma_{(t)}$ (relative to Σ).

Because any $\Sigma_{(s)}$ and $\Sigma_{(t)}$ that are statewise equivalent are state-ancestors of one another even when they are not identical, state-ancestory is not in general a partial order on the set of all Σ-segments. It is, however, coextensive with the Σ-ancestory relation, and hence likewise a partial order, over any subset of Σ-segments containing at most one Σ-segment from each statewise-equivalence class of Σ-segments. That is, state-ancestory determines a partial order on statewise-equivalence classes of Σ-segments.

Def. 5.22 requires neither that the input section $\Sigma_{(t')}$ nor the output section $\Sigma_{(t'')}$ of a Σ-segment $\Sigma_{(t)}$ have any special causal tie to the input section $\Sigma_{(t*)}$ of $\Sigma_{(t)}$, nor that $\Sigma_{(t*)}$ itself have any organizational cohesiveness. Stipulating that $\Sigma_{(t)}$ be Σ-compact not only tightens up $\Sigma_{(t)}$ but also forbids any Σ-explicit causal progression from events of $\underline{A}_{(t')}$ to events of $\underline{A}_{(t*)}$, or from events of $\underline{A}_{(t*)}$ to events of $\underline{A}_{(t'')}$, to be Σ-explicitly mediated by events not in $\Sigma_{(t)}$. However, this still allows $\Sigma_{(t')}$ and $\Sigma_{(t'')}$ to be coupled with $\Sigma_{(t*)}$ more loosely than we may consider appropriate in some analyses of system structure. Even when $\Sigma_{(t)}$ is Σ-compact, it is still possible for a Σ-explicit causal progression to leave $\underline{A}_{(t)}$ after starting in $\underline{A}_{(t')}$, to enter $\underline{A}_{(t'')}$ from outside of $\underline{A}_{(t)}$, or to go from $\underline{A}_{(t')}$ to $\underline{A}_{(t'')}$, without passing through $\underline{A}_{(t*)}$. We can easily-enough suppress these possibilities, if we wish to do so, by restricting our attention to "state-dominated" Σ-segments as follows:

<u>Definition 5.24</u>. Let $\Sigma_{(r)}$, $\Sigma_{(s)}$, and $\Sigma_{(t)}$ be any segments of <u>bcs</u> Σ. Then $\Sigma_{(s)}$ Σ-<u>mediates</u> between $\Sigma_{(r)}$ and $\Sigma_{(t)}$ just in case for every $\underline{C}$-chain $\underline{k}_1,\ldots,\underline{k}_n$ in Σ, if $\underline{k}_1$ is in the organizor $\underline{K}_{(r)}$ of $\Sigma_{(r)}$ while $\underline{k}_n$ Σ-matches an index in the organizor $\underline{K}_{(t)}$ of $\Sigma_{(t)}$, or if $\underline{k}_1$ is in $\underline{K}_{(t)}$ while $\underline{k}_n$ Σ-matches an index in $\underline{K}_{(r)}$, then at least one of indices $\underline{k}_1,\ldots,\underline{k}_n$ is in the organizor of $\Sigma_{(s)}$. (Note that this definition is satisfied vacuously if no index in $\underline{K}_{(r)}$ or $\underline{K}_{(t)}$ Σ-forecasts an index in the other.)

<u>Definition 5.25</u>. A segment $\Sigma_{(t)}$ of <u>bcs</u> Σ is <u>state-dominated</u> (relative to Σ) just in case (1) $\Sigma_{(t)}$ is Σ-compact; (2) the state section $\Sigma_{(t*)}$ of $\Sigma_{(t)}$ exists; (3) if either the input section $\Sigma_{(t')}$ or the output section $\Sigma_{(t'')}$ of $\Sigma_{(t)}$ exists, $\Sigma_{(t*)}$ Σ-mediates between the union of $\{\Sigma_{(t')},\Sigma_{(t'')}\}$ and any Σ-segment $\Sigma_{(s)}$ whose intersection with $\Sigma_{(t)}$ is null (i.e. that is disjoint with $\Sigma_{(t)}$); and (4) if both $\Sigma_{(t')}$ and $\Sigma_{(t'')}$ exist, $\Sigma_{(t*)}$ mediates between them.

I think that the structure envisioned by Def. 5.25 is in fact often presupposed in commonsense system concepts: in particular, it seems to be a necessary (though not sufficient) condition for Σ-segment $\Sigma_{(t)}$ to qualify as a "stage" of global system

Σ. It is not yet clear to me, however, whether state-domination has any deep theoretical significance beyond being a property that may well be entailed by other conditions we find useful to impose on Σ-segments worthy of special attention.

Let us now seek to formalize the classical conception of system <u>dynamics</u>. Commonsensically, this notion envisions that a temporally enduring global object consists of "stages" that are serially ordered by time, and that this object's causal condition at each stage of development is causally determined by the system's causal condition at a more-or-less just-preceding stage together with input to input to the global object that is essentially synchronic with the stage being determined. With this model in mind, let us say

<u>Definition</u> <u>5.26</u>. A segmented causal system $\Sigma_T^2 = \{\Sigma_{(t)}: \underline{t} \in \underline{T}\}$ is a <u>dynamic segmentation</u> of <u>bcs</u> Σ just in case, relative to Σ: (1) Each $\Sigma_{(t)}$ in Σ_T^2 is a Σ-compact, externally determined segment of Σ. (2) Σ is the union of Σ_T^2. (3) Σ_T^2 is linearly ordered by state-ancestory ~~relative to Σ~~ (i.e., the state sections, $\Sigma_{(s*)}$ and $\Sigma_{(t*)}$ respectively, of any $\Sigma_{(s)}$ and $\Sigma_{(t)}$ in Σ_T^2 exist, are distinct if $\Sigma_{(s)}$ and $\Sigma_{(t)}$ are distinct, and either $\Sigma_{(s*)}$ is a Σ-ancestor of $\Sigma_{(t*)}$ or $\Sigma_{(t*)}$ is a Σ-ancestor of $\Sigma_{(s*)}$). (4) For any distinct $\Sigma_{(s)}$ and $\Sigma_{(t)}$ in Σ_T^2 such that $\Sigma_{(s)}$ is a state-ancestor of $\Sigma_{(t)}$, the state section $\Sigma_{(s*)}$ of $\Sigma_{(s)}$ Σ-mediates between $\Sigma_{(t)}$ and any Σ-segment $\Sigma_{(r)}$ disjoint from $\Sigma_{(s*)}$ that is a segment of the union of all Σ-segments in Σ_T^2 which are state-ancestors of $\Sigma_{(s)}$. (5) For any $\Sigma_{(r)}$ and $\Sigma_{(t)}$ in Σ_T^2 such that $\Sigma_{(r)}$ is a state-ancestor of $\Sigma_{(t)}$, there exists a finite sequence $\Sigma_{(s_1)},\ldots,\Sigma_{(s_n)}$ of Σ-segments in Σ_T^2 such that $\Sigma_{(r)} = \Sigma_{(s_1)}$, $\Sigma_{(s_n)} = \Sigma_{(t)}$, and for each $\underline{i} = 1,\ldots,\underline{n}-1$, the state section of $\Sigma_{(s_i)}$ is Σ-immanent for the state section of $\Sigma_{(s_{i+1})}$.

The force of Def. 5.26's Clauses 1 and 2 is reasonably self-evident, except perhaps for stipulating that each $\Sigma_{(t)}$ in Σ_T^2 is externally determined. Although the definition of this in Def. 5.18 is not altogether optimal (it does not achieve its intended force for systems whose <u>C</u>-character contains laws with infinitely many input variables; and can perhaps better be replaced by a definition in terms of <u>C</u>-chains from which the present Def. 5.18 follows), it entails that for each $\underline{A}_{(t)}$ in each $\Sigma_{(t)}$ in Σ_T^2, all consequent events of $\underline{A}_{(t)}$ are determined by the set of all (local) input events of $\underline{A}_{(t)}$ according to regularities entailed by $\Sigma_{(t)}$'s $\underline{C}$-structure. Clause 2 can just as well be relaxed to allow Σ to be any intrinsic law-compositional extension of the union of Σ_T^2 so long as Σ in Clauses 1 and 3-5 is replaced by $U\Sigma_T^2$; but it is generally expedient to keep $U\Sigma_T^2$ as law-compositionally nonredundant as possible, insomuch as the more fulsomely Σ is an intrinsic law-compositional extension of $U\Sigma_T^2$, the greater the excursive thickness a Σ-segment in Σ_T^2 must have in order to qualify as Σ-compact in contrast to Σ_T^2-compact. It may also warrant mention that stipulating Σ-compactness for each $\Sigma_{(t)}$ in Σ_T^2 can be omitted from Clause 1, since this is in any case entailed by the remainder of Def. 5.26.

Clause 3 of Def. 5.26 is simple enough in its own right; but its main force is in conjunction with Clauses 4 and 5. By Clauses 3 and 5, for every $\Sigma_{(t)}$ in Σ_T^2 save possibly a first one there is at least one $\Sigma_{(s)}$ in Σ_T^2 whose state section $\Sigma_{(s*)}$ is Σ-immanent for the state section $\Sigma_{(t*)}$ of $\Sigma_{(t)}$. (Similarly, the state section of each $\Sigma_{(s)}$ in Σ_T^2 save possibly a final one is Σ-immanent for the state section of at least one other $\Sigma_{(t)}$ in Σ_T^2.) Then according to Clauses 3 and 4, if $\Sigma_{(s*)}$ is Σ-immanent for $\Sigma_{(t*)}$ ($\Sigma_{(s)}$ and $\Sigma_{(t)}$ in Σ_T^2), then for every Σ-object $\underline{A}$, every Σ-explicit causal progression entering $\underline{A}_{(t)}$ from elsewhere in $\underline{A}$ passes through one or more events common to $\underline{A}_{(s*)}$ and $\underline{A}_{(t*)}$--which in light of Clause 1 entails that all events of $\underline{A}_{(t)}$ are wholly determined by events of $\underline{A}_{(s*)}$ together with the (global) input events of $\underline{A}$ under Σ that are also input events of $\underline{A}_{(t)}$. Clause 4 also entails that each $\Sigma_{(t)}$ in Σ_T^2 is state-dominated, which together with Clause 1 entails that all events of the output section $\underline{A}_{(t'')}$ of $\underline{A}_{(t)}$, if extant, are wholly determined by events of $\underline{A}_{(t*)}$ together with any (global) input events of $\underline{A}$ under Σ that are also input events of $\underline{A}_{(t)}$.

Finally, it is important to appreciate that Clauses 1 and 5 of Def. 5.26 do <u>not</u> require Σ_T^2 to be discretely ordered by state-ancestory. It is entirely permissible under Def. 5.26 that between any two Σ-segments in Σ_T^2 lies an infinitude of others.

Footnotes, Section 5.

[1]Simultaneous infinite compositions, though demanding of technical care, are essentially unproblematic conceptually. Successive infinite compositions (notably backward, to preserve a given set of output events) is something else again. These can be well-defined in very special cases, but in general present enormous conceptual difficulties. (See Rozeboom, 1978, Section III.)

[2]For superior connotations, $\alpha\underline{A}$ might be called the "constitution" of $\underline{A}$ But this locution is so unwieldly that I shall use "core" instead.

[3]The nature of L-analyzability can also be clarified as follows: The organization $\langle\psi,\phi\rangle$ of any bcs Σ uniquely determines a function $\delta = \{\delta_j : j \in J\}$ from $\underset{\sim}{A}$ into $\underline{J}$-indexed locus sets, where each $\underline{j}$ in $\underline{J}$ is a pair comprising an index $\underline{k}$ in Σ's organizor and the indices of locus-positions in $\underline{F}_k$'s scope, such that for each $\underline{A}$ in $\underset{\sim}{A}$, $\delta\underline{A}$ is a $\underline{J}$-indexed set of loci in $\underline{A}$'s core in which, however, the the same locus may occur more than once. Then Σ is L-analyzable just in case, for every $\underline{j}$ and $\underline{j}'$ in $\underline{J}$, $\delta_j\underline{A} = \delta_{j'}\underline{A}$ is true for one $\underline{A}$ in $\underset{\sim}{A}$ only if it is true for all. If this identity condition is satisfied, an L-analyzor can be constructed from δ by restricting $\underline{J}$ to exclude redundancies. This also suffices to satisfy Clause 4 of Def. 5.3.

[4]That α is unique up to *locus isomorphism for a given L-analyzable bcs follows from the construction partially sketched in fn. 3.

[5]This follows just from the formalized properties of Σ-ancestory; it does not require appeal to causality's axiomatic (HT-4) partial-order character. Even so, the latter is required to make the formal definition of Σ-ancestory a useful system concept.