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# Nicod's Criterion: Subtler than You Think

In a recent note, Horwich (1978) challenges the foundations of Hempel's classic paradox of confirmation by a clever example purporting to show that under Nicod's Criterion, data can be made to confirm a hypothesis with which they are logically incompatible. Specifically, Horwich observes that '$Pb$' (i.e., 'object $b$ has property $P$') is formally equivalent to '$(x)(\sim Px \cdot \sim Pb \supset x \neq b)$. The latter has form '$(x)(\psi x \supset \phi x)$' with '$\sim P\_\_\_\_ \cdot \sim Pb$' for $\psi$ and '$\_\_\_\_ \neq b$' for '$\phi$', while the observation that distinct objects $a$ and $b$ both lack $P$, i.e. that $\sim Pa \cdot \sim Pb \cdot a \neq b$, can be expressed as '$\psi a \cdot \phi a$' for these same instantiations of the predicate markers. Accordingly, if an uncertain generality '$(x)(\psi x \supset \phi x)$' were always to be confirmed by an observation of form '$\psi a \cdot \phi a$', as Nicod's Criterion has long been presumed to say, then we could confirm that $b$ has $P$ by observing that $b$ and some other object both lack $P$—a flagrant absurdity.

Horwich's example is a useful one, because it makes stunningly evident that something is fundamentally wrong with the traditional reading of Nicod's Criterion. Horwich does not himself seek to locate the difficulty; but it is important for that to be done, since exposed thereby are certain elementary but seminal confusions about the confirmation of conditionals that still pervade the literature on Hempel's Paradox.

Nicod's own statement of his criterion, slightly paraphrased from the passage quoted in translation by Hempel (1945), is that "a fact consisting of the presence of $Q$ in a case of $P$ is favorable to the law '$P$ *entails* $Q$,' whereas if it consists of the absence of $Q$ in a case of $P$ it is unfavorable to this law." Or more briefly, omitting its uncontroversial second clause,

$N$: That all $P$s are $Q$s is confirmed by the presence of $Q$ in a case of $P$.

When formalizing this notion, however, it is important to be clear that one proposition does not confirm/disconfirm another *simpliciter*, but does so only relative to a body of antecedent information. (Any reader who honestly disagrees on this point is excused from further participation in this exercise.) Once the relativity of confirmation is made explicit, we can see that Nicod's Criterion is importantly ambiguous or, more precisely, is subject to misinterpretation in that respect.

Let $\beta$ be our background knowledge, while $Pa \cdot Qa$ is our new evidence regarding hypothesis

$H$: (x)(If $Px$ then $Qx$).

(I deliberately avoid writing $H$ as '$(x)(Px \supset Qx)$,' for presuming the connective in lawlike generalities to be material implication begs basic questions.) Then one way to interpret $N$ is

$N_1$: Given $\beta$, $H$ is confirmed by $Pa \cdot Qa$.

This is Hempel's construal of Nicod's Criterion and the one that has prevailed in the ensuing literature, including Horwich's note. But $N$ can also be explicated as

$N_2$: Given $Pa$ and $\beta$, $H$ is confirmed by $Qa$.

Which of these, $N_1$, or $N_2$, is closer to Nicod's intent? Surely there can be little question that $N_2$ is the correct reading; otherwise, we should regard $N$ as equivalent to

$N'$: That all $P$s are $Q$s is confirmed by the presence of $P$ in a case of $Q$.

The operational difference between $N$ and $N'$ is profound: $N$ directs us to test hypothesis $H$ by producing or searching out $P$s and ascertaining whether they are all $Q$s, whereas $N'$ directs us to produce or search out $Q$s and ascertain whether they are all $P$s. And whereas $N_2$ clearly sides with $N$ as opposed to $N'$, $N_1$ is indifferent between $N$ and $N'$.

But if the intuitive plausibility of Nicod's Criterion resides in $N_2$, it should come as no surprise to find that mistaking this as support for unqualified $N_1$ has mischievous consequences. For the degree to which $H$ is confirmed / disconfirmed by $Pa{\cdot}Qa$ given $\beta$ is a function jointly of the degree to which $H$ is confirmed / disconfirmed by $Qa$ given $Pa \cdot \beta$ *and* the degree to which $H$ is confirmed / disconfirmed just by $Pa$ alone given $\beta$. In cases where $Pa$ alone is confirmationally irrelevant to $H$ given $\beta$, as has surely been the tacit assumption underlying failure in virtually all the past literature on Hempel's Paradox to distinguish between $N_1$ and $N_2$, $Qa$ confirms $H$ given $Pa{\cdot}\beta$ just in case $Pa{\cdot}Qa$ confirms $H$ given $\beta$. But, our intuitions for most commonsense form-$H$ generalities notwithstanding, it is *not* true either that Nicod's Criterion is committed to

$N^*$: Given $\beta$, $Pa$ alone is confirmationally irrelevant to hypothesis $H$,

or that $N^*$ is a principle holding for all or even many choices of $P$ and $Q$ in $H$. Horwich's example is a good case in point, for his particular $Pa$ totally disconfirms his particular $H$. But more generally, the persistent enigma of Hempel's Paradox has been obscurity in the force of $H$'s connective inflated into incoherence by failure to observe that violations of $N^*$ keep $N_1$ and $N_2$ from being coextensive.

More precisely, the surface strain of Hempel's Paradox lies in our feeling that datum $\sim Pa \cdot \sim Qa$ ought not to matter for $H$, even though this is a positive instance of $(x)$(If $\sim Q$ then $\sim Px$) and hence apparently confirms both the latter and its *prima facie* contrapositive equivalent $H$. However, $\sim Pa \cdot \sim Qa$ is intuitively irrelevant to $H$ only if $\sim Pa$ is; and $\sim Pa$ is intuitively irrelevant to $H$ just in case the same is true of $Pa$. So the root of Hempel's Paradox is above all intuition $N^*$, albeit that intuition would have been much less seductive had 'If $Px$ then $Qx$' been sincerely interpreted as "Either not-$Px$ or $Qx$," nor would its presumption have gone undetected had the difference between $N_1$ and $N_2$ been appreciated. For without conflation of $N_1$ and $N_2$ to overgeneralize the confirmatory force of positive instances, it would not have seemed so evident that $\sim Pa \cdot \sim Qa$ should always confirm the contrapositive of $H$.

If we accept the modern orthodoxy that diachronic confirmation is best modeled by conditional credibility in a synchronically coherent belief system, or agree at least provisionally that $p$ confirms $q$ given $\beta$ in an ideal belief system just in case the credibility of $q$ given $p \cdot \beta$ exceeds the credibility of $q$ given $\beta$, the interplay between confirmation of $H$ by $Pa$ given $\beta$, by $Qa$ given $Pa \cdot \beta$, and by $Pa \cdot Qa$ given $\beta$, can be made completely clear with remarkable simplicity. Specifically, if $Pr(p \mid r)$ is the credibility (subjective probability) of proposition $p$ conditional on information $r$, and we use the "Confirmation Ratio"

$$CR(q, p \mid r) =_{\text{def}} Pr(q \mid p \cdot r)/Pr(q \mid r)$$

to appraise whether $p$ confirms $q$ (if $CR > 1$), disconfirms $q$ (if $CR < 1$), or is indifferent to $q$ (if $CR = 1$) given information $r$, it is elementary to show (Rozeboom, 1971) that for any propositions $h$, $p$, $q$, $r$ having nonzero unconditional credibilities,

$$CR(h, p \cdot q \mid r) = CR(h, p \mid r) \times CR(h, q \mid p \cdot r).$$

Hence in particular, if $H$ continues to be the conditional generality that all $P$s are $Q$s, the confirmation of hypothesis $H$ by observation $Pa \cdot Qa$, given any background information $\beta$, is

$$CR(H, Pa \cdot Qa \mid \beta) = CR(H, Pa \mid \beta) \times CR(H, Qa \mid Pa \cdot \beta).$$

In its proper reading, Nicod's Criterion claims only that $CR(H, Qa \mid Pa \cdot \beta) > 1$; and in a coherent belief system this is indeed true for any admissible interpretation of the conditional in $H$ so long as $Qa$ remains uncertain given only $Pa \cdot \beta$, i.e., unless $H$ does not really matter in $Pr(Qa \mid H \cdot Pa \cdot \beta) = 1$. Meanwhile, $N^*$ has the force $CR(H, Pa \mid \beta) = 1$; so in cases where $N^*$ holds we have

$$CR(H, Pa \cdot Qa \mid \beta) = CR(H, Qa \mid Pa \cdot \beta) \qquad \text{(given } N^*\text{)}.$$

Thus were conditional generalities always confirmationally indifferent to instantiations of their antecedents, $N_1$ and $N_2$ would be equivalent and Hempel's Paradox would indeed be paradoxical. But once it is clear that $CR(H, Pa \mid \beta)$ can be sufficiently less than 1 to make data $Pa$ and $Qa$ jointly indifferent to or disconfirmatory of $H$ given $\beta$ despite $Qa$'s mitigation of $Pa$'s disconfirmatory import for $H$, there is little more to say about Hempel's Paradox in its original formulation except to bury it with the epitaph that its vast literature rests upon confusion.

Even so, the *source* of this confusion has major import, via indifference condition $N^*$, for the logic of conditionality and its role in inductive inference. What is remarkable about $N^*$ is not that it fails as a universal generalization but that intuition is so strongly disposed to accept its natural instances, i.e., excluding artifices like Horwich's construction, even though $N^*$ is demonstrably not tenable (Rozeboom, 1971) if the 'If ... then ...' connective in $H$ is taken to be material implication. Roughly speaking, to believe $N^*$-wise for some particular $H$ and natural $\beta$ is tantamount to presuming that any causal / becausal linkage between $P$-hood and $Q$-ness in object $a$ passes from $Pa$ to $Qa$ rather than from $Qa$ to $Pa$ or to $Pa$ and $Qa$ jointly from a common source. The details of this situation are rather complex; and while I would like to think that Rozeboom (1971) is more than a bare beginning, much remains for development, starting with a conception of nomic structure more articulate than what has yet appeared in the philosophical or scientific literature. (In work still unpublished, I have been able to carry this much farther than the simplistic schema in Rozeboom, 1971.) It also remains to explicate contingency concepts that can take part in rational instances of $N^*$ for natural $\beta$, and to explore how our *uncertainties* about nomic structure are most properly manifested in our conditional credences.

Meanwhile, my present objective is to plead again for recognition of the real issues behind Hempel's Paradox. What is important to learn from this is not that lawlike generalities are not always confirmed by their positive instances, or that confirmation / disconfirmation of theories by data is strongly relative to our background knowledge and the specifics of the theory at issue. (That is true enough, but to declaim it is fatuous unless we can also say something useful about what does determine the confirmational import of particular data for particular theories in particular contexts.) Neither is it that the paradox vanishes if one formulates the problem in terms of conditional credibilities. (Many writers have taken that approach without recognizing that the foundational question at issue is what confirmation structure should / do we in fact put into our credibility distribution over combinations of $H / {\sim} H$, $Pa / {\sim} Pa$, and $Qa / {\sim} Qa$ given realistic background knowledge.) The point is not even that Nicod's Criterion is unassailable when correctly understood. (The trouble with $N_2$ is not that it is problematic, but that the near triviality of its truth is too closely akin to that of the "hypothetico-deductive" model of scientific inference, the pernicious vacuity of which diverts attention from the operational problems of theory adjudication— Rozeboom, 1970, p. 93ff and 1972.

The seminal insight to be reclaimed from the debris of Hempel's Paradox, the gold that remains in the crucible after its dross has burned away, is recognition and eventual understanding of the profound communalities underlying our inductive reasoning and our suppositions about the world's nomic order. (See also Spielman, 1969.) No account of confirmation in which causal / becausal concepts lack prominence, or which cannot explain why intuition should insist so adamantly on frequent instances of $N^*$, can claim much relevance to our real-world inferential behavior. And no theory of lawfulness and conditionality that slights the view of *de re* contingency inescapably latent in our judgments of confirmational irrelevance, as manifested paradigmatically but far from exclusively in $N^*$, can hope to capture the force of conditional connectives in science and everyday life.

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